‘Unfretting the violin’

Theory, composition and practice of microtonal string music

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Abstract

This dissertation presents my personal research in the field of microtonal string music and an exploration of the performer as a theoretician and composer through four case-studies, the inspiration of which is founded in historical sources. Case-study I introduces my interaction with quarter-tone material as a *performer.* The problems encountered in Case-study I are the origins of the exploration through the *performer-composer-theoretician* symbiosis in Case-study II. Case-study III presents a free and creative attempt to write a larger work which explores areas discussed in Case-study I and II, and is the result of my collaboration as a *performer-composer* with another composer. While Case-study I and II have a cause-effect relationship, Case-study III was developed independently, but previous reading of Case-study II is necessary. Case-study IV presents a contrasting example with an unsystematic approach to microtonality, in which my role as a *composer* is limited: the measure of the microtone is dictated by the performers involved and uses a new compositional technique which I have termed 'discrete-glissando'.

The result of my artistic research is presented as a music book accompanied by a 'homemade' CD with the intention of reaching, inspiring and encouraging composers and performers to go beyond their roles.

Contents

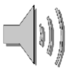
Supporting material

CD (attached to the dissertation's back cover)

1. *Quarter-tone Study n.1* by Thanakarn Schofield *
2. *Quarter-tone Study n.2* by Thanakarn Schofield *
3. *Quarter-tone Study n.3* by Thanakarn Schofield *
4. *53TET Study n.1* by Sara Cubarsi *
5. *53TET Fugue* by Sara Cubarsi *
6. *53TET 2-Part Counterpoint* by Sara Cubarsi **
7. *Seven Microtonal Variations* by Thanakarn Schofield and Sara Cubarsi *
8. *Microtonal Spiral* by Sara Cubarsi **
9. *8-Part Intonation Study* by Sara Cubarsi **

* Sara Cubarsi, violin

** London/Nashville Exchange Project 2014 ensemble

 indicates that the figure or score shown in the dissertation can be found in the CD

Microtonal Studies

- *Three Quarter-tone Studies* for solo violin by Thanakarn Schofield
- *53TET Study, 53TET Fugue* for solo violin and *53TET 2-Part Counterpoint* for string ensemble by Sara Cubarsi
- *Seven Microtonal Variations* for solo violin by Thanakarn Schofield and Sara Cubarsi
- *Microtonal Spiral* and *8-Part Intonation Study* for ensemble by Sara Cubarsi
- Appendix

Introduction

In the classical contemporary music world there is a recurring attempt to establish strong collaborative relationships between composers and performers. In the past, figures such as Bach and Mozart were recognized for their talents both as performers and composers. Later, possibly because of the increasing challenging technical demands of the repertoire, the figure of the performer-composer started to fade away and strong collaborations between individual great performers and composers emerged, such as Brahms' with clarinettist Richard Muehlfeld who played an integral part in the composition of the Sonata in F minor.[1] The public acknowledgement and recognition of such relationships was kept private during a period when well-known performers mostly gave recitals of dead composers.[2] Collaborations such as Brahms' are studied by scholars in the search for a model of the collaborative process.[3] Recent models of composer-performer collaborations can be found between John Cage and pianist David Tudor, Luciano Berio and singer Cathy Berberian,[4] and David Gorton and violinist Peter Sheppard-Skaerved, for example.

Not so trendy is the collaboration between theorists, composers and performers and rarer is the integrated figure of the theoretician-composer-performer which this dissertation explores in order to show the ways in which this alliance can contribute to the contemporary music scene.

The disengagement between theorists and musicians takes us back to the 1850s when, with the dissemination of the equally tempered piano, the figure of the piano tuner emerged as a separate entity to that of the musician.[5] Harry Partch (1901-1974) is perhaps the most recent well-known model of a composer, performer,

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1 Emily Tyndall, "Johannes Brahms and Muhlfeld: Sonata in F Minor for Clarinet and Piano Op. 120 No. 1", (PhD diss., CSU, Schwob School of Music), accessed 09/2014, http://www.archive.org/stream/johannesbrahms00tynd/johannesbrahms00tynd_djvu.txt.

2 Lukas Foss mentions Arthur Schnabel's lifelong collaboration with Schoenberg which 'was kept scrupulously to himself' while Schnabel continued to perform Beethoven and Schubert. Lukas Foss, "The Changing Composer-Performer Relationships: A Monologue and Dialogue", *Perspectives of New Music*, Vol. 1, No. 2 (Spring, 1963), 46.

3 Fabrice Fitch and Neil Heyde, "'Recercar'- The Collaborative Process as Invention", *Twentieth Century Music*, 4/1 (Cambridge University Press, 2010), 71-95.

4 Foss, "The Changing Composer-Performer Relationships: A Monologue and Dialogue", 46.

5 Hermann L. F Von Helmholtz (1885), *On the sensations of tone as a physiological basis for the theory of music* (3rd Ed., London: Longmans, Green, and Co., 1895), 548-549.

theorist and instrument maker.[6] This integrative figure can also be found in composers such as James Tenney[7] who use computers, theorize on technological matters to serve their compositional ideas, and programme computers to 'perform' their music. In terms of performance practice, microtonality[8] is a perfect playground for the application of tuning theories. Although technology has certainly facilitated the practical study of different tuning systems, this project only addresses microtonal composition in the field of acoustic music, working within the (limitless) possibilities of human perception.

Contemporary violin methods include surprisingly little tutoring on microtonality. Quarter-tones appear (very briefly) in Borciani's *Per la Musica Moderna e contemporanea*[9] and in Rostal's edition of Carl Flesch's *Scale System,*[10] perhaps as a result of microtonal inflections that appeared in late-romantic violin repertoire by Ysaye, Bartók and Enescu.[11] However, a large part of the microtonal repertoire bases its tuning in intervallic relations within the overtone series, continuing the work of Renaissance theorists who were involved with the

6 The study of the partnership between theoreticians, composers, performers and instrument makers is another world to explore, particularly in the field of microtonality and wind instruments. Projects that have explored the relationship between instrument building and specific tuning systems have been undertaken by the Microtonal Trumpet project and by oboist Christopher Redgate. This is not such an issue for string instrument makers, since unfretted instruments can play any pitch within their range and the use of microtones can be achieved through aural training. However, string instruments with longer fingerboards (such as the viola) are better suited for microtonal explorations, which is why Partch's new string instruments have lengthened fingerboards. "The Microtonal Trumpet", accessed 09/2014, http://www.microtonaltrumpet.com. Michael Hooper, "The start of performance, or, does collaboration matter?", *Tempo*, Volume 66, Issue 261 (Cambridge University Press 2012). Harry Partch*, Genesis of A music* (Da Capo Press, Second Edition, 1974).

7 Jean-Claude Risset, "About James Tenney, Composer, Performer, and Theorist", *Perspectives of New Music*, Vol. 25, No. ½ (Winter, Summer, 1987), 549-561.

8 I particularly like Marc Sabat's definition of microtonality as 'an approach to pitch which acknowledges the musical possibility of this entire glissando-continuum and is not limited to the conventional twelve equal tempered pitch-classes'. "New Music Box", accessed 09/2014, http://www.newmusicbox.org/articles/intonation-and-mirotonality.

9 Paolo Borciani, *Per la Musica Moderna e Contemporanea: 20 9 esercizi di tecnica superior violinistica: simboli grafici* (Ricordi, 1977), Exercise n. 172, 42.

10 Carl Flesch, *Scale System: Scale Exercises in All Major and Minor Keys for Daily Study* (Ed. Max Rostal, 1970), 129-128.

11 See Ysaye's Sonata n.3 for solo violin (dedicated to Georges Enescu), Enescu's Sonata n.3 for violin and piano and Bartók's Sonata for solo violin (Urtext Edition). Bartók wrote two microtonal sections in the last movement of his Solo Sonata with an alternative 12-tone version. Yehudi Menuhin (the dedicatee) exchanged letters with Bartók during the composition of the work and always played the alternative version. The original version has remained (in Nordwall's words) as a 'curiosity'. This is further evidence of the reservation with which violinists approach (and avoid approaching) microtones. Ove Nordwall, "The Original Version of Bartók's Sonata for Solo Violin", *Tempo: New Series*, No. 74 (Autumn, 1965), 2-4.

tuning of keyboard instruments before equal temperament took over.[12] The obscure unpublished work *Les Duresses*[13] by Marc Sabat was written to assist violinists in the study of just intonation. The poor presence of a practice code is proof of the disengagement between microtonal composition and performance.[14]

Having surveyed the microtonal repertoire for unfretted instruments, I have also witnessed a lack of understanding (or laziness to do so: the fingerboard is not a keyboard!) of the theoretical and psychological implications of some microtonal structures by composers, who usually either ask for inhumane precision or for inharmonic pitch relationships that can't be intuitively reached by the string player.

My project proposes a way into microtonality which I will argue is friendly and intuitive from a performer's perspective, provides renewed tonal[15] material from a composer's perspective and is systematic from a theoretical viewpoint. Parts 1 and 2 give historical and repertorial context for the material I will be presenting later. Part 3 opens up the discussion of intonation and psychology in music performance with a practical case-study of the quarter-tone. For this part, I requested the collaboration of composer Thanakarn Schofield with whom I have worked before and whose lyric writing I love. Upon collaboration proposal he wrote (but here without my collaboration yet) the *Three Quarter-tone Studies.* My unsympathetic discourse about equal temperament is counterbalanced by my search for a methodology in the practice of these studies. The

12 Artusi's position is somewhat similar to mine. In the excerpt below he criticizes composers using inharmonic interval relations that from the point of view of his current state of music theory and tuning practices produced discrepant sounds and difficulties in its performance. I particularly like his reference to lutenists, who are able to temper the scale using frets: 'All this disorder stems from nothing other than that, they understand nothing other than that which their caprices tell them will be all right. For they sometimes set us out intervals which they themselves do not know, and say that they are something new even though they are older than the cuckoo-bird: like the following, the first of which, they say, is neither a 6th nor a 7th [C#-Bb], but resounds very well to their ears, which are purged. Intervals false for singing; but for playing on lutes, good. [...] the natural voice is not suited to negotiate such unnatural intervals by means of natural ones, not having a preset stopping place like an artificial instrument. […] It cannot justly divide the whole tone into two equal parts […] two things should be considered here: […] first that the inventors of such ill-conceived facts […] will never be able to divide the 81:80 proportion into two parts with known and specified rational numbers, [and] secondly, that it is impossible that the residuum of [each of] the two semitones would amount to half of the 9:8 whole-tone. For it is a very firm conclusion in mathematics that no superparticular proportion can be divided into two equal parts with known and rational numbers [...] .' Mark Lindley, "Chromatic Systems (Or Non-Systems) from Vicentino to Monteverdi", review of "Theories of Chromatic and Enharmionic Music in Late 16th Century Italy" by Karol Berger, *Early Music History*, Vol. 2 (1982), 402-404.

13 "Les Duresses", accessed 09/2014, http://www.marcsabat.com/pdfs/LD.pdf.

14 Musicians who are members of the so-called Historically Informed Performance movement are well aware of the multiplicity of historical temperaments. Such performers are excluded from my criticism of the prevailing lack of interest in tuning theories among performers.

15 My system is tonal because it is derived through fifths, which represent the relationship tonic-dominant-subdominant.

discussion continues over in Part 4 where, in reply to the quarter-tone system, I will use three studies (*53TET Study*, *53TET Fugue* and *53TET 2-Part Counterpoint*) to propose how a 53-tone equal tempered scale can be used as a departing point for microtonal experimentation in a performer-friendly way. This part of the commentary is particularly directed to composers and shows the value of the performer's intellectual involvement in the compositional process. Part 5 presents a piece which resulted from the creative collaboration with Schofield and is inspired by the microtonal sources presented earlier in the project. Our relationship was not only as performer-composer but also as composer-composer. Part 6 presents a radically different approach to microtonality – less systematic and more contemplative – with two case-study pieces by myself for ensemble (*Microtonal Spiral* and *8-Part Intonation Exercise*). Although pitch frequency is a crucial element in these studies it is not part of an intonational system, but can, however, be considered microtonal. I invented the 'discrete-gliassando' technique which consists in the alternation of unison and microtonal clashes between two or more performers where they dictate the relative size of each clash.

All the ideas presented here work as a sample of the possible outcomes of such an interlinked process. A brief CD with the studies mentioned above is provided as testament of the value of my research, together with a separate booklet with the (provisional) final scores of the studies in order to provide an overview of the creative outcome of this project, potentially interest other artists and become expanded.

Part 1. Microtonal composition and its precedents

A general definition for microtonal composition is that in which the tuning system or pitches used transcend the equally tempered range and therefore has arisen as a movement of its own after the standardization of equal temperament. A more selective definition would be that in which the tuning or temperament is the matter of exploration and distinctive characteristic of a work. For the latter denotation there are two major trends. One bases its principles in pure tunings or Just Intonation[16] (henceforth JI), the first advocates of which (in Europe) can be traced back to the Ancient Greeks. The other is a side effect of the establishment 12-Tone Equal Temperament (henceforth ET or TET) as a regular sampling of simple ratio scales since the 19th Century.[17] This division will serve as a general framework for this section and will be used to describe the origins of different microtonal perspectives in new music together with a selection of representative figures. As will be explained, not all strictly microtonal composers fall into one of these groups, and not all composers who use microtones can be categorized as microtonal composers.

1.1. The battle of intonation

In Western music history, JI is known to have already been used as a basis for harmony by scientists and musicians who followed the pioneering work of Pythagoras (c. 540-510 B.C.) in Ancient Greece. However, construction of ratio scales in other cultures predates that of Ancient Greece. The oldest evidence of all known comes from Ancient China.[18] Nonetheless, there is little evidence of continental exchange regarding music

16 The term 'Just Intonation' will be used to describe 'a system in which interval – and scale – building is based on the criterion of the ear and consequently a system and procedure limited to small-number ratios [ratios between a sound's harmonics]; the initial interval in Just Intonation is 2/1, and stemming from this are the wealth of musical intervals inherent in small-number tonal relationships. Just Intonation is a generic term describing this procedure'. Partch, *Genesis of A music*, 71.

17 Helmholtz, *On the sensations of tone as a physiological basis for the theory of music*, 548-549.

18 More than two thousand years before Pythagoras, Ling Lun was building instruments out of bamboo pipes (presumably a sequence of different sizes of pipes stuck together) arbitrarily divided into 81 unities. The pipes would be of five different sizes with the relative ratios of 1/1, 3/2, 9/8, 27/16 and 81/64, giving a 5-tone antecedent to the 12-tone Pythagorean scale that was later developed by the same Chinese theoretician by adding one more pipe (or *lü*) and another set of 6 *lüs* transposed up a chromatic semitone, giving a

theory.[19] Pythagoras has ever since left a permanent mark in European music. The so-called Pythagorean scale was built by adding and subtracting pure fifths. Its divisions were demonstrated on a single stringed instrument (namely the monochord).[20] The Ancient Greeks used the term tetrachords to describe groups of four notes that had an interval of a perfect fourth between its extremes and, depending on how the ratios were distributed,[21] it was in either the diatonic, chromatic or enharmonic genera[22] – the diatonic containing a semitone, the chromatic a third tone and the enharmonic a quarter tone step approximately. Different tunings of the tetrachord were explored after Pythagoras by figures such as Archytas (c. 400 B.C.), Aristoxenus (c. 330 B.C.), Eratosthenes (276-196 B.C.) and later on by Ptolemy (2nd Century A.D.).[23] Ptolemy substituted all the complex ratios from the Pythagorean divisions of the monochord (derived through fifths) by the simplest ratios of equivalent sounding intervals.[24] Keeping the octaves and fifths pure has continued to be an almost mythical goal (not always achieved and sometimes intentionally compromised) of performers, theoreticians and instrument builders in Europe.

The expansion of the Roman Church spread the practice of monodic plainchant[25] and evidence from the High Middle Ages proves that singers used a monochord with Pythagorean divisions to learn to intonate the chants.[26] Even though Pope John XXII continued to impose the use of 'concords such as the octave, fifth and fourth, that enrich the melody and may be sung above the simple ecclesiastical chant'[27] in 1326 – with the development of vocal music from monody to polyphony, the growth of instrumental and secular music, the development of notation and the appearance of the composer's authorship – the Pythagorean divisions of the octave were slowly left aside in favour of the theory defended by Ptolemy (or what is commonly referred to as

cycle of 12 fifths transposed into a one-octave scale. Partch, *Genesis of A music*, 362, 401.

19 Partch recalls tracing Pythagoras' influences in Indian music as a one-off transcontinental exchange: 'I even recall seeing a Hindu book on musical theory which stated that Pythagoras visited India, bringing the Harmonical Proportion to its presumably benighted musicians.' He is also known to have been in Egypt, Chaldea and Babylon. Partch, *Genesis of A music*, 363.

20 Ibid., 363, 398-406.

21 As David Creese points out "one of the consequences of expressing intervals as ratios is that it is impossible to express exact semitones and quarter-tones: there is no ratio of numbers which will divide the ratio of the tone equally" David Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge University Press, 2010), 19. Partch, *Genesis of A music*, 366.

22 Creese, *The Monochord in Ancient Greek Harmonic Science*, 18.

23 Ibid., 22-50. Partch, *Genesis of A music*, 364-368.

24 Partch, *Genesis of A music*, 369.

25 Paul Griffiths, *A Concise History of Western Music* (Cambridge University Press, 2006), 12.

26 Guido D'Arezzo, *Micrologus* (Ed. Waesberghe, American Institute of Musicology, 1955).

27 Ian Johnston, *Measured Tones: The Interplay of Physics and Music* (3rd Ed., CRC Press, 2009), 18.

Historical JI, derived through adding and subtracting pure fifths and thirds from a generating tone, which ultimately resulted in the acceptance of thirds as consonant intervals[28] [29]) that became the basis of harmonic writing and instrument building until less than two centuries ago.

New levels of complexity in the music of the *Ars Nova*[30] included the development of keyboard instruments.[31] This led to an increased interest in continuing the study the tuning systems left handed down by the Ancient Greeks. Music performance demanded new adjustments (or compromises), i.e., temperaments,[32] in the tuning of fixed pitch instruments. Performers of unfretted instruments were expected to adjust to the tunings of the less flexible ones.

On the one hand, musicians had been dealing with the intonational possibilities offered by ratio scales and on the other the problematic presence of the comma prevailed.[33] The Syntonic comma or the comma of

28 Intervals that may have sounded 'imperfect' to the ear, derived through other than 2/1 or 3/2 ratios, were avoided. The Pythagorean major third of ratio 81/64 – as demonstrated on the monochord that served to train singers during the High Middle Ages as explained by Guido D'Arezzo in his Micrologus – was perceived to be more discordant than the pure 5/4 and was treated accordingly in terms of voice leading. It wasn't until later that was observed by Walter Odington (c. 1240-1280) that singers would intuitively adjust thirds to a simpler ratio and with the development of polyphony the consonant third in JI became into compositional use. Odington also left one of the earliest evidence of the term 'major' triad. Guido D'Arezzo, *Micrologus.* Partch, *Genesis of A music*, 372.

29 According to C.W. Leverett Johnson, the earliest reference to consonance and dissonance was made by Euclid (c. 3rd Century B.C.), consonance being 'the blending of a higher with a lower tone' and dissonance the 'incapacity to mix, when to tones cannot blend, but appear rough to the ear.' Partch, *Genesis of A music*, 367. Johnson, *Musical Pitch and the Measurement of Intervals Among the Ancient Greeks*, 50-51.

30 In 1316, Philip de Vitry published the treatise 'Ars Nova' which describes the principles of the new polyphonic and florid style that emerged in the Renaissance. However, the treatise still uses Pythagorean descriptions of intervals, which were soon to be replaced by the Ptolemaic scale, and later by a variety of mean-tone temperaments. Ian Johnston, *Measured Tones: The Interplay of Physics and Music*, 18. Partch, *Genesis of A music*, 372.

31 Their predecessor is the hydraulic organ which already existed in the third Century B.C.. It's very likely that no more than two keys were pressed at once since they were widely spaced. Their use disappeared by the 8th Century, but by the 10th Century they were rediscovered and placed in churches. Claviers seem to have been used in the Middle Ages, they only contained the diatonic keys and B-flat, but by the 14th Century they had gained the five accidentals. Laurence Libin, "Keyboard Instruments", *The Metropolitan Museum of Art Bulletin: New Series*, Vol. 47, No. 1 (Summer, 1989), 5.

32 Bartolomeus Ramis de Pareja (1482) has been considered the first 'temperer' (or almost so), about who Harry Partch writes: ' "We have made all our divisions very easy" wrote Ramis of his division of the monochord. These easy divisions were actually certain just ratios – 5/4 and 10/9 for example – substituted for the accepted Pythagorean tuning which was "tiresome for the singers and irksome for the mind". This hardly constitutes temperament, either in fact or in attitude, but the break with Pythagoreanism had been made, and by the close of the next century there is hardly a musica practica that does not present theories tooled by the "practical" hand of adulteration.' Partch, *Genesis of A music*, 374.

33 'The difference between two between-degree relationships'. Partch, *Genesis of A music*, 368.

Didymus[34] is the difference in intonation between a scale degree arrived at through the cycle of fifths and the same degree arrived at through a cycle of pure thirds. The other (unpopular) excess was the Pythagorean comma of ratio (3^12)/(2^19), the amount by which the twelfth fifth in a 3/2-cycle surpasses the generating tone.

In the 16th-century Italy there was a blossoming of innovative approaches to keyboard instruments building that consisted in adding extra keys order to accommodate a wider number of chords in JI[35] as a consequence of the reluctance to accept an equally tempered version of all scales and determination to avoid harmonies (referred to as 'wolf' tones[36]) distorted by the Syntonic comma.[37] Musicians came up with different solutions including a variety of mean-tone temperaments (i.e., tempering two whole tones in order to get a just third in spite of false fifths).

These seemingly impractical solutions for keyboard instruments with too many keys and the presence of wolf-tones led to the alternative solution: the tempering of a cycle of pure fifths by the Pythagorean comma, giving birth to 12-tone Equal Temperament[38] [39] which in 1856 became the standard tuning, first applied to piano

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34 Didymus (1st Century A.D.) gave his name to the excess of an 81/80 between the ratios 9/8 and 10/9 present in the diatonic tetrachord of the Ancient Greek genera. Partch, *Genesis of A music*, 368.

35 The *cembalo cromatico* may have preceded the well-known *archicembalo* attributed to Nicola Vicentino – probably dating from before 1537. Cristopher Stembridge, "The Cimbalo cromatico and Other Italian Keyboard Instruments with Nineteen or More Divisions to the Octave", *Performance Practice Review*, Vol. 6, no. 1 (1993), 33-59.

36 'The perfectness of the thirds was then the chief consideration with musicians, the consequence being that the fifths were caused to be very flat, and some were left so intolerably bad that their beatings could only be compared to the howling of "rampacious wolves", and many poor jokes were perpetrated by old organists at their expense.' Hermann Smith, *The Art of Tuning the Pianoforte* (Robert Cocks & CO, Modern Methods, No.7), 15.

37 Vicentino (c. 1550) built his *archicembalo* with 31 tones to the octave and re-explored the enharmonic Greek genera. Numerous experiments of this sort took place in 16th- and 17th-century Italy, including Zarlino's diagrams of a 24-tone keyboard from his *Istitutioni harmoniche* (1558) and the *Clavicymbalum universale*, *seu perfectum* described by Praetorius in 1660 which had 19 tones per octave and seven possible transpositions. Partch, *Genesis of A music*, 377. Stembridge, "The Cimbalo cromatico and Other Italian Keyboard Instruments with Nineteen or More Divisions to the Octave", 39.

38 Marin Mersenne (1588-1648) gave the first recorded tables in Europe for the 12 equal divisions and in 1688 Art Schnitger tuned the first organ to Marsenne's divisions in Hamburg. Partch, *Genesis of A music*, 382, 384.

39 Although recent research indicates that Bach did not use equal temperament for his *Well-Tempered Clavier* – which uninhibitedly moves through the 24 major and minor keys – it is perhaps a forecast of its inevitability. With the standardization of figured bass and modulating harmonies composers started accepting the idea of a new tuning compromise. John Barnes argues that he would have used an unequal temperament with wide major thirds on white keys originally advocated by Werckmeister (1645-1706). John Barnes, "Bach's Keyboard Temperament: Internal Evidence from the Well-Tempered Clavier", *Early Music*, Vol. 7, No. 2 (Apr. 1979), 236-249. Herbert Anton Kellner, "A propos t'une réimpression de la Musicalische Temperatur (1691) de Werckmeister", *Revue de Musicologie: Échelles Musicales: Modes Et Tempéraments*, T. 71, No. 1/2 (1985), 184-187.

manufacturing by Broadwood & Sons.[40]

1.1.1. Representation of JI lattices

A JI lattice is a visual representation of a tuning system where all pitches are derived through prime-number partials, i.e., each pitch can be calculated through the addition or subtraction of a simple ratio.[41] Each generating ratio (which designates relationships within the overtone series) is represented in one dimension. The three main tuning systems discussed above are the Pythagorean, the just and the mean-tone. The former two can be described as systems in 3-limit JI and 5-limit JI respectively and the mean-tone is a temperament.

In a 3-limit lattice the generating interval is the second overtone or third partial (3/2) and the lattice is therefore one-dimensional:[42]

Ab	Eb	Bb	F	C	G	D	A	E
128/81	32/27	16/9	4/3	1/1	3/2	9/8	27/16	81/64

In a 5-limit lattice the generating intervals are the major third (5/4) which corresponds to the fifth partial (vertical axis) and the previous prime-number partial, the fifth (horizontal axis), and is therefore represented in two dimensions.[43]

C	G	D	**A**	**E**	**B**	**F#**	C#	G#
Ab	Eb	Bb	**F**	**C**	**G**	**D**	A	E
Fb	Cb	Gb	**Db**	**Ab**	**Eb**	**Bb**	F	C

40 Helmholtz, *On the sensations of tone as a physiological basis for the theory of music*, 548-549. Ian Johnston, *Measured Tones: The Interplay of Physics and Music*, 74.

41 Note that adding and subtracting intervals is a multiplicative operation.

42 The Pythagorean comma constitutes the difference between two enharmonically equivalent tones in the fifths cycle, e.g. B# and C, where B#=3/2·3/2·(…) 12 times, i.e., (3^12)/(2^12). This gives us a pitch 19 octaves higher than the original C which to the C octave range is (3^12)/(2^19). Therefore, the Pythagorean comma is (3^12)/(2^19).

43 The Syntonic comma constitutes the difference between two 'same' notes of the first and the second rows above or below.

Or in ratios:

160/81	40/27	10/9	**5/3**	**5/4**	**15/8**	**45/32**	135/128	405/256
128/81	32/27	16/9	**4/3**	**1/1**	**2/3**	**9/8**	27/16	81/64
312/405	256/135	64/45	**16/15**	**8/5**	**6/5**	**9/5**	27/20	81/80

A selection of these pitches would constitute the tuning of a keyboard in *Historical JI*[44] depending on the key signature. The 12 pitches shown in bold above would work well in a C-major context, but if modulating to G major, V/V would contain a wolf-tone, the chord being 9/8 – 45/32 – 5/3 instead of 9/8 – 45/32 – 27/16.[45]

In a JI lattice there will be as many dimensions as generating (always just) intervals. In mean-tone temperament, the aim is to preserve the thirds as pure as possible, despite imperfect fifths altered by ¼ of a Syntonic comma.[46] Figure 1.1 compares the size of the tone and semitone in the systems discussed.

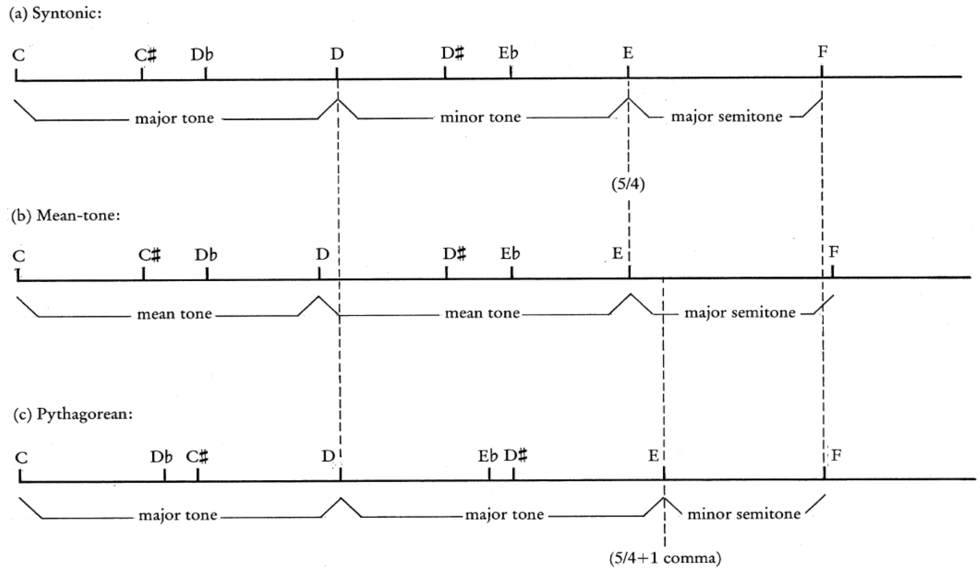

Fig 1.1. Patrizio Barbieri's graph showing different sizes of tones and semitones in JI, mean-tone temperament and Pythagorean intonation.[47]

44 As discussed in Part 1.1, *Historical JI* does not imply mean-tone temperament but refers to a tuning based on the principles advocated by Ptolemy, i.e., a selection of 5-limit ratios.

45 The distance between the 27/16 and the 5/3 As is 80/81, the Syntonic comma.

46 'If we call this fraction 't' [a fraction of the Syntonic comma, e.g. 1/4], then the amounts by which the major 3rds and 6ths are tempered will be 1-4t [in the case of ¼-comma mean-tone the thirds and sixths will remain pure] and 1-3t respectively. If the major 3rd were tempered 7/11 comma (as in equal temperament), the diminished 4th […] would be, as it happens, the same.' Lindley, "Chromatic Systems (Or Non-Systems) from Vicentino to Monteverdi", 380.

47 Patrizio Barberi and Sandra Mangsen, "Violin intonation: a historical survey", *Early Music*, Vol. 19, No. 1 (Oxford University Press, Feb. 1991), 71.

1.1.2. The revival of JI in new music

Preceded by the gap filled with the omnipotent 12TET, number of (mostly American) composers started re-exploring just tunings using higher partials to generate more intervals and harmonies in the 20th Century. Such composers are considered microtonalists, but their interest comes from 'getting the music in tune',[48] rather than adding smaller or different subdivisions of 12 to the octave (or other intervals). Music had been written in different tunings for ratio scales in, 3-limit and 5-limit JI, so it seems like an unavoidable progressive step to experiment extending the limit to higher partials (i.e. 7-limit, 11-limit, etc.). This also has its precedents – such as Tartini who tried to make musical use of the seventh harmonic[49] – but it was not welcomed by their contemporaries.[50]

Harry Partch (1901-1974) uses a 43-tone scale. His scale is generated through 11-limit JI lattices, i.e., adding just fifths (3/2), thirds (5/4), sevenths (7/4) and elevenths (11/8) to a fixed tone, which in his case was always G.[51] He also built his own instruments, including 43-tone keyboards, adding strings to instruments of the harp family and lengthening the fingerboard or changing the size of the box of the violin family for example. Ben Johnston (born in 1926) became his tuner and student and also invented a new notation system for JI tuning which extended to and included the 31st partial.[52] In his case, the reference point was a fixed C major scale in 5-limit. LaMonte Young's (b.1935) music also uses Extended-JI lattices and it embraces the comma as a musical

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48 Ben Johnston in Douglas Keilsar, "Six American Composers on Nonstandard Tunings", *Perspectives of New Music*, Vol. 29, No. 1 (Winter 1991), 200.

49 Giuseppe Tartini (1692-1770) attempted to include the 7th harmonic in musical use, demonstrating its notation and function in his *Trattato di musica* (Padua, 1754) stating: 'This harmonic 7th is not dissonant but consonant […] it has no need either of preparation or of resolution: it may equally well ascend or descend, provided that its intonation be true.' Barberi and Mangsen, "Violin intonation: a historical survey", 84. Partch, *Genesis of A music*, 388. Matthew Shirlaw, *Theory of Harmony* (Novello & Co., 1917), 208. The scientist Leonhard Euler (1707-1783) is also recognized for having embraced the 7th harmonic as a musical interval. According to F. J. Fétis 'he has been the first to see that the character of modern music resides in the chord of the Dominant Seventh, and that its determining ratio is that of number 7.' Partch, *Genesis of A music*, 388. Shirlaw, *Theory of Harmony,* 348.The seventh harmonic has been popularized more recently as a colouring note (namely the 'blue seventh') specially by singers and brass players in jazz music.

50 Jean Adam Serre's point seems to be after the fact than 12TET does not embrace an acceptable approximation of the 7th harmonic: 'It is not surprising that the theory which, so to speak, drowns the commas and the quartertones in the modern temperament, and moreover banishes from harmony, […] the sound expressed by the ratio 4:7, as a false and non-harmonic sound – it is not surprising, I say, that this theory gives us no enlightenment as to the origin and the possibility of the enharmonic of the Ancients.' Partch, *Genesis of A music*, 387.

51 Partch, *Genesis of A music*,195-258.

52 Keilsar, "Six American Composers on Nonstandard Tunings", 178-179.

interval in his six hour long *The Well-Tuned Piano* among other pieces.[53] Lou Harrison (1917-2003) is an example of a composer who approached JI from the perspective of ethnic instruments from Asia, such as the gamelan, and favours those scales originally found in JI to which Western instrumentalists easily adjust.[54]

In Europe we find the 1970s spectral movement[55] which driven by acoustical research (and the use of computers) can be compared to that of the American JI in that the overtone series provides a framework for musical structure. Romanian composer Horatiu Radulescu (1942-2008) is usually considered a spectralist (although in his writings he claims not to be!) and has written string music including very high partials accessed as natural harmonics in *Das Andere* for solo viola and his 5th String Quartet *Before the Universe was born* for example.

1.1.3. Extended-JI lattices

An example of an Extended-JI lattice will include the seventh harmonic and needs the addition of a new dimension. The vertical axes represent intervals derived through 5/4, the horizontal 3/2 and the depth 7/4 (Figure 1.2).

53 Michael Harrison (Young's piano tuner and composer), personal communication (Atlantic Music Festival, Waterville, 2014).

54 Keilsar, "Six American Composers on Nonstandard Tunings", 178.

55 'One of the most basic changes introduced by spectral composers was the generation of harmonic and timbral musical structures based upon frequential structures. […] For spectral composers, microtones are not the result of scales built on frequency ratios, nor even one of tuning. Instead, the microtones in spectral music are simply approximations of a set of frequencies to the nearest available musical pitches.' Joshua Fineberg, "Spectral Movement: history and techniques", *Contemporary Music Review 2000*, Vol. 19, Part 2 (Overseas Publishers Association, 2000), 81-113.

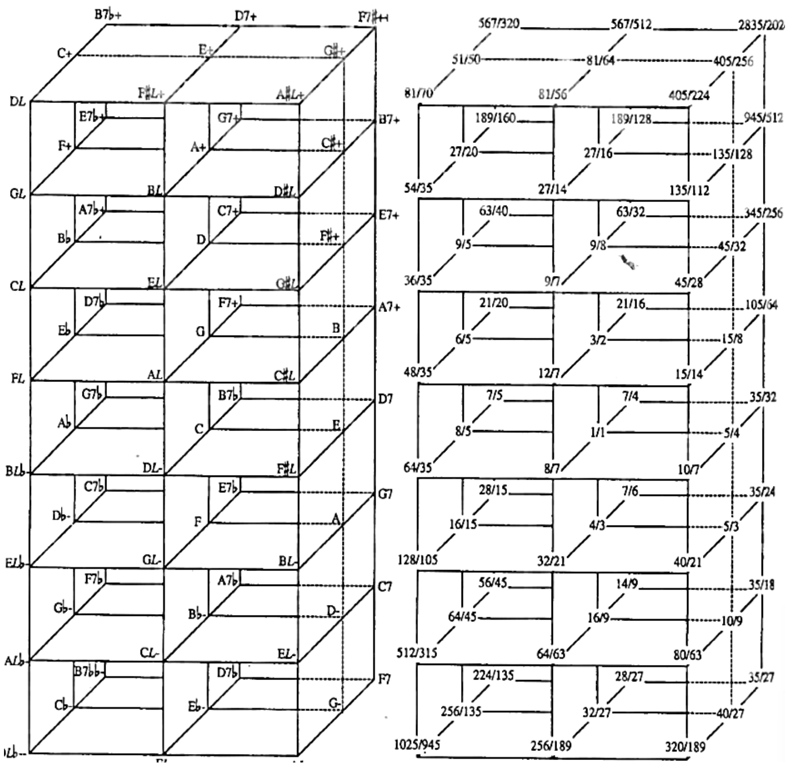

Fig *1.2.* Johnston's 7-limit lattice.[56]

1.2. Equal Temperament

Once again it was the Chinese who first calculated the exact (to nine decimal places) value of string lengths tuned to 12TET in the 4th Century, as well as the formula from which the scale is derived in 1569.[57] However, their system was predominantly theoretical and in everyday musical practice they continued to play using their justly tuned instruments.[58] Not long after the establishment and acceptance of equal temperament in Europe, Schoenberg was writing music in twelve-tone rows. Could this be interpreted as a rejection of tonality? Or rather as a consequence of the incapabilities of equal temperament? As Lou Harrison puts it: 'Schönberg wisely observed there was no tonality in it.'[59]

Alois Hába (1893-1973) was an admirer of Schöenberg and his serial technique, resulting in 12 limited pitch classes, and divided the octave into other multiples of 12 (and later of 3 or 4). He wrote a treatise about

56 Ben Johnston, *Maximum Clarity and other writings on music* (Urbana, University of Illinois Press 2006), 72.

57 Partch, *Genesis of A music*, 369, 381.

58 Ibid., 381.

59 Keilsar, "Six American Composers on Nonstandard Tunings", 198.

quarter-tone, third-tone and sixth-tone music theory.[60] Hába's efforts to establish a microtonal school in Europe were first interrupted by the Nazi movement both in Germany and later in his country, the former Czechoslovakia, by Stalin and the Communist Party.[61] In contrast, Ivan Wyschnegradsky's (1893-1979) interest in microtones came from a more philosophical perspective and wanted to get hold of more tones in the *sound continuum.*[62] [63] His work and research was also abruptly disrupted in 1942 and captured by the Nazis but he was released shortly thereafter.[64]

Other composers who have left their musical footprints in the history of ETs (multiple of 12) are Charles Ives, Easly Blackwood, William Schottstaedt, Julián Carrillo and Joe Maneri. Ives (1874-1954) was already writing his Chorale for Strings in Quarter-tones in 1913.[65] Later he met Hans Barth, German composer, who was working with quarter-tone pianos in New York and also became aware of Hába's work. However, his quarter-tone pieces are a very small portion of his entire body of work.[66] The Mexican Julián Carrillo (1875-1965) wrote a music theory book called 'Sonido 13' and divided the octave into 96 parts and also built extended harps and guitars.[67] Blackwood (born 1933) uses ETs up to 24 notes per octave and organizes them in modes, octatonic or whole-tone scales depending on the musical style he is after.[68] He works almost entirely with synthesizer. Schottstaedt (born 1951) also uses computers to generate microtonal music and favours the use of ETs as well

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60 Alois Hába, *Neue Harmonielehre des diatonischen, chromatischen, viertel-, drittel-, sechstel-, und zwöftel-tonsystems* (1925).

61 "Tonalsoft", accessed 08/2014, http://www.tonalsoft.com/monzo/haba/haba-worklist.aspx.

62 "Association Ivan Wyschnegradsky", accessed 08/2014, http://www.ivan-wyschnegradsky.fr/en/biography/.

63 While searching for quarter-tone pianos during the 1920s he met Alois Hába. He established himself in Paris and attracted other composers' attention such as that of Messiaen, in the Festival of quarter-tone music (1937, Paris). Ibid.

64 Ivan Wyschnegradsky, *La Loi de la pansonorité* (1953*), Une philosophie dialectique de l'art musical* (1936), *L'ultrachromaticisme et les espaces non octaviants* (1972) among others. Ibid.

65 His motivation came from the fact that his father Georges Ives had a 'Quarter-tone machine' at home and used to play melodies and harmonies in quarter-tones to the family who would then sing them back (even though it sounds more like a family game, apparently he used it as a means of punishment!). Later he met Hans Barth, German composer, who was working with quarter-tone pianos in New York and also became aware of Hába's work. James Wood, "Microtonality: Aesthetics and Paracticality", *The Musical Times*, Vol. 127, No. 1719 (June 1986), 328.

66 Ives wrote "Some Quarter-tone Impressions" in 1924-25 where he explains the structures of hexachords, his music sounds rather colouristic and ultrachromatic. Howard Boatwright, "Ives's Quarter-Tone Impressions", *Perspectives of New Music*, Vol. 3, No. 2 (Spring - Summer, 1965), 22-31.

67 "Julian Carrillo y el Sonido 13", accessed 08/2014, http://www.sonido13.com.

68 Keilsar, "Six American Composers on Nonstandard Tunings", 177.

as Pythagorean tuning. He has used up to 144 divisions per octave,[69] which partly explains why he advocates computer-performed music!

These composers were perhaps not so concerned by the fact that 24 or 48 divisions of the octave do not improve equally tempered falsities and provide the ear with even greater harmonic complexities. Within equally tempered systems other musicians, such as the Dutch Adriaan Fokker (1887-1972),[70] searched for better compromises than 12TET dividing the tone in an uneven number of steps.

Joe Maneri (1927-2009) composed and taught an aural class in 72TET in New England Conservatory of Music from the 1970s, which is now taught by his former pupil Julia Werntz. He approaches microtonal music from a Schenkerian and voice-lead perspective[71] and, having been a jazz musician and therefore accustomed to the seventh harmonic, he was also interested in the JI possibilities within 72TET. Maneri and Fokker could be classified as an 'overlapping' trend interested in a variety of ETs in order to access certain JI ratios.

1.3. The overlap

John Eaton (1935) is an American composer who uses quarter-tone music but expects performers to inflect microtones differently depending on voice-leading and musical context.[72] Mandelbaum uses 31TET to approximate JI intervals including the 7th and 11th partials. He expects his music to be played in just intonation,

69 Ibid., 179.

70 He was interested in including an acceptable intonation for the seventh harmonic in keyboards, very well approximated by 31-tone equal temperament (therefore having 5 steps within a tone) and found in countless folk melodies. Joseph Yasser, also from the Netherlands, had a similar approach using a 19-tone scale explained in his *A Theory of Evolving Tonality*. "Huygens-Fokker Foundation", accessed 06/2014, http://www.huygens-fokker.org/docs/fokkerorg.html. Joseph Yasser, *A Theory of Evolving Tonality* (Da Capo Press Inc., Facsimile of 1932 ed., Dec 1975).

71 Julia Werntz, "Adding pitches: Some New Thoughts, ten years after Perspectives of New Music's 'Forum: Microtonality Today' ", *Perspectives of New Music*, Vol. 39, No.2 (Summer 2001), 161.

72 'Most of my pieces involve quarter-tones, but I'm not after rigorous twenty-four-tone equal temperament. Quarter-tones become a field of action for performers. […] Performers will inflect the pitches as the music requires, depending for example on the position of the note in a phrase. They already do so in chromatic music. This is the whole point of music-making; you're not working in a vacuum, but with performers who are very sensitive. […] Every instrumentalist and singers can hear a quarter-tone as a distinct pitch. […] The other thing that performers can hear is just intervals, where the beating is removed.' Keilsar, "Six American Composers on Nonstandard Tunings", 177.

and, for him, 31TET is a handy pitch sample to get around unusual intervals on a keyboard while composing.[73] Ezra Sims and James Tenney also use unpair divisions of the tone such as 19TET or 31TET as practical solutions to approach 11-limit JI harmonies.[74]

1.4. Other

There is a large group of (mostly European) composers who are not advocates of a 'new' or 'better' tuning or temperament (and therefore may not be classified as strictly microtonal composers) but have incorporated microtones in their music very succesfully. In *Mikrotonalitäten* (1999), composer Georg Friderich Haas classifies them as composers interested in 'music based on beating and detuning phenomena' such as Xenakis or Scelsi, and 'music based on irrational/inharmonic structures' such as Cage or Lachenman.[75] Other composers are Gloria Coates whose work not only includes a great deal of quarter-tone based music, but also makes extensive use of glissandi, Pendercki and Ligeti who often used quarter-tones as an effect or as clusters, and the British school of complexity, which includes composers such as Michael Finnissy and Brian Ferneyhough among others.

73 Ibid., 179.

74 "New Music Box", accessed 09/2014, http://www.newmusicbox.org/articles/intonation-and-microtonality/.

75 Ibid.

Part 2. The microtone: morphology and syntax

The choice of notation is (or should be) dependent on tuning principles and the nature of the instruments in question. This choice is nonetheless pre-conditioned by the state of the modern 'classical' musical ear. Given the development of the 7-note name system, the 5-line stave notation and perfect pitch (especially as a consequence of the universality of concert pitch), most microtonal notation is comprised of varying versions of the flat and sharp accidental signs.[76]

Musicians from Ancient Greece, plainchant scribes from the Middle Ages and 17th-century Italians came up with symbols to indicate the diesis and the raising or lowering of a comma, but this will be omitted here.[77] Musical examples are selected given my general definition of microtonal composition which includes microtonalists who were not advocates of a specific tuning and excludes anything written before the piano 'invaded' our homes. Since then, new symbols for intervals smaller than the equally tempered semitone appeared along with other notation signs to indicate what before was taken for granted (or wanted, i.e., consonant ratios) at the same time as expanding the limits of JI. However, individuals such as Partch or Radulescu have invented a new notation system better suited to their music which completely allows them to detach form the 12-tone system.[78]

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76 The importance of the 'microtonal structure' above its symbology is ignored in Gardner Read's classification of microtonal notations in *20th Century Microtonal Notation*. As Rudolf Rasch points out in his review of the book, Read spends a hundred pages showcasing hundreds of different notations for various divisions of the equally tempered semitone, and leaves little place to explain notations that are subject to other microtonal structures that don't include the pitches of 12TET. This is proportional to the occurrence of microtonal notation Read has come across and does not offer a balanced overview of how different systems have been notated and why. Gardner Read, *20th Century Microtonal Notation*, Contributions to the Study of Music and Dance, Number 18 (Westport, Connecticut: Greenwood Press, 1990). Rudolf Rasch, "Review of Gardner Read, 20th Century Microtonal Notation*", Perspectives of New Music*, Vol. 29, No. 1 (Winter, 1991), 258-262.

77 Hoyle Carpenter localizes the use of the symbol 'X' as 'half of the less half-note' or diesis, as Thomas Morley (1557/58-1602) does in *A Plaine and Easie Introduction to Practicall Musicke,* and found in a manuscript dating from 1559 found in Kastner's "Los manuscritos musicales núms. 48 y 242 de la Biblioteca General de la Universidad de Coimbra", Anuario Musical, (1950) 78ff. Hoyle Carpenter, "Microtones in a Sixteenth Century Portuguese Manuscript", *Acta Musicologica*, Vol. 32, Fasc. 1 (International Musiciligical Society, Jan. - Mar., 1960), 23-28. Lindley, "Chromatic Systems (Or Non-Systems) from Vicentino to Monteverdi", 377-404.

78 'Music is composed on the basis of instruments, and instruments are individual. What is rational and well-integrated for one is quite the opposite for another. Symbols should represent, for the player, physical acts upon the strings […] or whatever vibratory bodies he

This section illustrates this process, as well as the ideology behind the choice of notation through a selection of musical excerpts.

2.1. Notation in ETs

The quarter-tone is probably the most widely used microtone and is certainly welcomed by those performers accustomed to 'thinking' in 12TET. This can be explained by the fact the piano is used in elementary music education for the development of aural skills. Quarter-tones thus take the form of a 12TET scale transposed by a quarter-tone above or below and use variations on the traditional accidental signs. Hàba uses 𝄲 , 𝄰 , 𝄳 and 𝄭 to indicate a quarter sharp three-quarter sharp, quarter flat and three-quarter flat, whereas Wysnegradsky uses 𝄲 , 𝄰 , 𝄳 and 𝄭 respectively.

Even though the use of pitch classes is very practical in the field of ETs, particularly for analytical purposes, Julian Carrillo went a step further by replacing the traditional notation with pitch class notation, believing that this would make reading the score 'as easy as reading the newspaper'. [79]

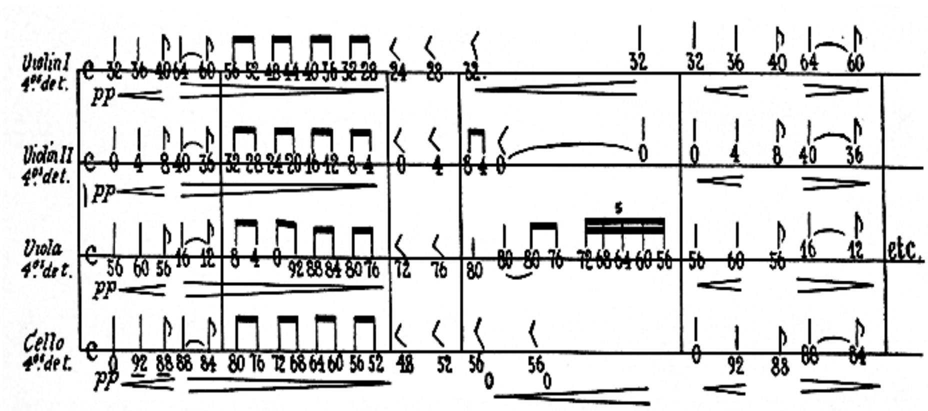

Fig 2.1. Excerpt from *Balbuceos* by Julián Carrillo with 96 pitch classes.[80]

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has before him, but they do not represent such acts very well unless the peculiarities of string patters […] are taken into account as the basis for the figuration of the symbols.' Partch, *Genesis of A music*, 198.

79 Miles L. Skinner, "Toward a Quarter-Tone Syntax: Selected Analyses of Works By Blackwood, Hába, Ives, and Wyschnegradsky" (PhD. diss, University of Buffalo, 2006), 11.

80 "Julian Carrillo y el Sonido 13", accessed 09/2014, http://sonido13.com/nuevaescrituramusical2.html.

Fig *2.2*. Carrillo's transcription of *Balbuceos* in traditional notation.[81]

While Wyschnegradsky and Ives integrate microtones into their chordal structures, Hába's microtones don't generally interfere with the tonal structure of the piece. Wyschnegradsky created a musical language of his own and his unique microtonal scale is built on a pattern that does not repeat itself after an octave.[82] Ive's *Quarter-tone Chorale* (1913-1914) demonstrates that quarter-tones systematically appear on the thirds and seventh of traditional triads.[83]

Blackwood and Hába use microtones within tonal conventions, using triads and seventh chords as structural pillars.[84] Figure 2.4 shows Skinner's Schenkerian approach to Hába's music. The microtones have an ornamental function. This process can also be applied to analyze the music by Blackwood or Joe Maneri.

Fig 2.3. Hába's String Quartet n.3, 3rd Mvt., mm. 7-14.[85]

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81 Ibid.

82 Wyschnegradsky, *Ultrachromatisme et éspaces non-octaviants* (1972).

83 Patricia Strange and Allen Strange, *The Contemporary Violin: Extended Performance Techniques* (University of California Press, 2001), 74.

84 Skinner, "Toward a Quarter-Tone Syntax: Selected Analyses of Works By Blackwood, Hába, Ives, and Wyschnegradsky", 39-42.

85 Ibid., 38.

Fig 2.4. Skinner's sketch for Figure 2.3.*[86]*

A different example of the use of quarter-tones within a melody can be found in Xenakis' *Mikka* and *Mikka 'S'* for solo violin (Figure 2.5). Although quarter-tone symbols shape the melodic contour of the pieces and the static points, the fact that they are linked through a glissando suggests that the idea he is after is not a discrete 24TET scale but expresses the capability of the violin to play an infinite continuous line getting a higher resolution of the edges of this *continuum* through quarter-tones. The fact that he omits enharmonics suggests that his quarter-tones should be played as equally tempered as possible. Simon Holt has a somewhat similar approach in a 12-tone context, leaving aside the flat accidentals in his *Two Movements for String Quartet.* The score does not include performance notes, suggesting that it is up to the performers to interpret the tunings of the intervals.[87]

Fig 2.5. Excerpt from Xenakis' *Mikka* for solo violin.[88]

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86 Ibid.

87 Curiously this is what Artusi argued (although he contradicted himself, see footnote 12) stating that 'in song [...] the major 3rd is always to be divided into two-whole tones, one larger, the other smaller [...]. It is for the singer to judge which of them he prefers to execute below or above as the other parts and the melody itself are seen to require'. Lindley, "Chromatic Systems (Or Non-Systems) from Vicentino to Monteverdi", 391.

88 Iannis Xenakis, *Mikka* (Ed. Salabert, 1972).

Ton de Leeuw is known for his equal division of intervals such as the minor third or major second,[89] a variant of the ET trend. In his String Quartet n. 2, the performer is instructed to divide the minor third into 4 intervals of 75 Cents and the major second into 3 intervals of 67 Cents. These microtonal notes are left without note-heads in the score. For example, between the first F and D on the first violin part, the performer should play an F lowered by 75 Cents, an E lowered by 50C and an E♭ lowered by 25C (Figure 2.6).

Fig 2.6. Ton de Leeuw's String Quartet n.2.[90]

2.2. Notation in Extended-JI

The most varied and imaginative notation systems for microtonal composition are found in the JI field. Some composers have used ratios to indicate the intonation of an interval, sometimes in reference to a fixed pitch or scale or only to the previous note. Others have chosen to write standard notation with or without microtonal alterations. I will give four illustrative examples of this variety from work by Partch, Radulescu and Johnston.

The following excerpt is from Partch's 'A Dream' from *Eleven Poems by Li Po* (1930-1933). The total number of tones per octave used in this piece is 37, all of which have a simple ratio relationship to G, which in Partch's music always represents 1:1.[91] He built an 'adapted' viola with a fingerboard extended by six inches whose strings are tuned in perfect fifths. The ratios in the score indicate the interval between a note and G,

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89 Strange and Strange, *The Contemporary Violin: Extended Performance Techniques*, 77-78.

90 Ibid.

91 Ibid., 153.

therefore the first note in the excerpt represented by 6/5 is a B-flat slightly higher (16 Cents) than that of 12TET.[92]

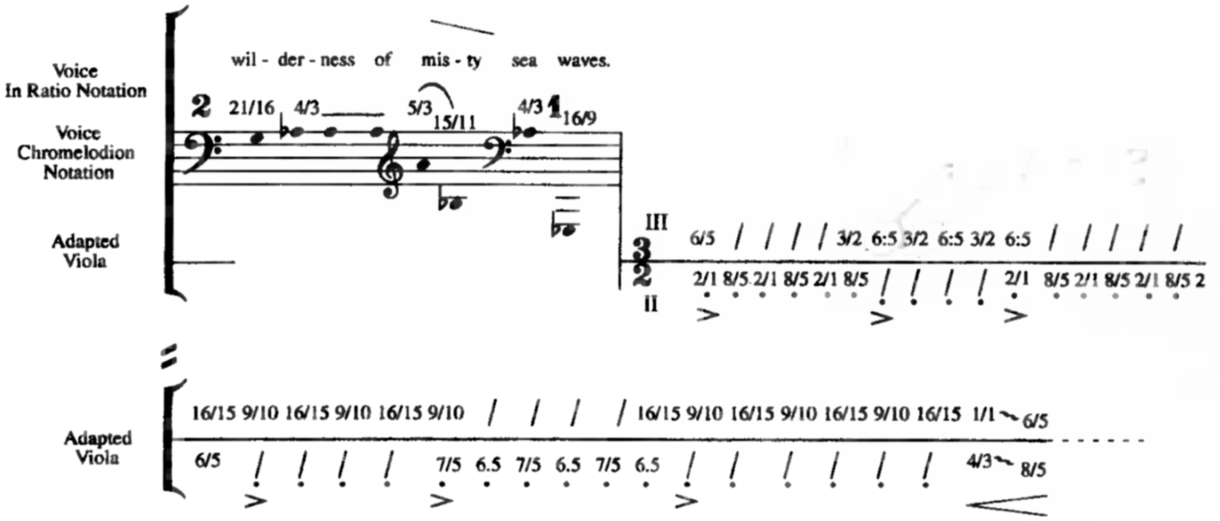

Fig 2.7. Excerpt of 'A Dream' by Harry Partch.[93]

The excerpt below is form Radulescu's *Das Andere* (originally for solo viola, but it can be played on violin or cello too). The four lines indicate the strings from high to low and the numbers indicate the partials played as natural harmonics. The effects of the right hand and bowing are indicated by the other symbols and explained in his performance notes.

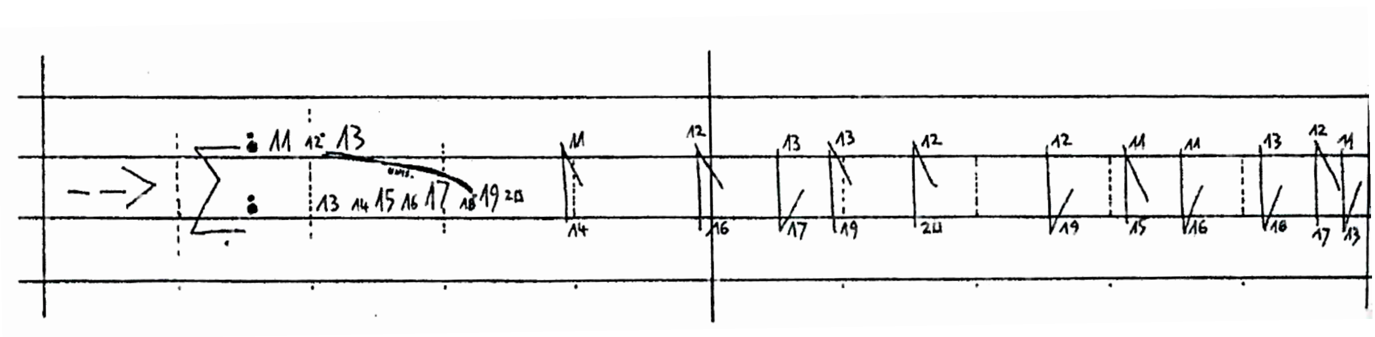

Fig 2.8. Excerpt from *Das Andere* by Horatiu Radulescu.[94]

92 The score includes a chromelodeon part written using 5-line stave tablature notation, so that if the singer plays the key-name indicated, it will play back the required pitch out of the 37 tones. This is for use in rehearsals only. The octave range is indicated by a number at the beginning of each new passage. This is an accessible example of the early stages of the development of Partch's system. Other composers such as Lydia Ayers have used ratio notation as a complementary tool to conventional notation with microtonal accidentals. Strange and Strange, *The Contemporary Violin: Extended Performance Techniques*, 153-155.

93 Ibid.

94 Horatiu Radulescu, *Das Andere* Op.49 (Lucero Print, 1983).

In Johnston’s music, microtonal accidentals alter the pitch so that the ratios between the notes are equivalent to any of the ratios accessed through the partials up to the 31st, always with reference to a justly tuned C major scale. The Syntonic comma is indicated with a ‘+’ or ‘–’. For example, an open G-string played on the violin is usually tuned two fifths below A (1/1 : 3/2 : 3/2 = 4/9, or 9/8 below A), but the uninflected G from the C major scale is 10/9 below A. Thus if Johnston wants to indicate the open G-string he will write ‘G –’.

Most of Johnston's microtonal symbols describe the ratio they relate to. For example, an F7 will be altered by 49 Cents with respect to the ¾ F from C, becoming the seventh partial of G– . The inflection provided by the 11th partial is indicated by an arrow up or down and it is very close to the tempered quarter-tone.

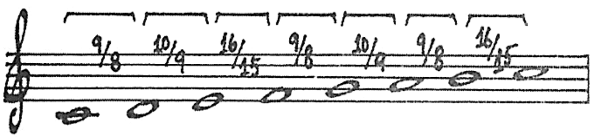

Fig 2.9. 5-limit JI C major scale.[95]

Fig 2.10. Johnston's notation for the violin's open strings tune in pure fifths. [96]

Figure 2.12 shows an excerpt from his fourth string quartet. It has a theme and variations structure on the hymn ‘Amazing Grace’, and each variation is centred in the exploration of a different harmonic or tuning system. The opening is in Pythagorean tuning. The B is left unaltered on a ‘G –’ major harmony, making the third wider than that in 5-limit JI (G/B is represented by 5/4, and G–/B by 81/64). The harmonies get progressively more complex in each variation. This notation also allows the combination of symbols, for example: a ‘♭’ and a ‘–’, or a ‘7’and a ‘–’ (see the second bar of the first violin part or the third bar of the second violin part).

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95 Johnston, *Maximum Clarity and other writings on music* , 78

96 Ibid., p.80.

raise	lower	ratio	cents	amount by which...	exceeds
♯	♭	25/24	71	5/4	6/5
+	−	81/80	22	9/8	10/9
L	7	36/35	49	9/5	7/4
↑	↓	33/32	53	11/8	4/3
13	ƐI	65/64	27	13/8	8/5
17	ㄥI	51/50	34	17/16	25/24
6I	19	96/95	18	6/5	19/16
23	Ɛᄅ	46/45	38	23/16	45/32
29	6ᄅ	145/144	12	29/16	9/5
31	IƐ	31/30	57	31/16	15/8

Fig 2.11. Notation symbols in Johnston's system.[97]

Fig 2.12. Johnston's String Quartet no. 4, beginning of the fifth variation.[98]

On the other hand, Lou Harrison's music is often written with no new codification and includes gamelans. In his Suite for Violin and American Gamelan the violinist is supposed to adjust to the just tuning of the gamelan by ear.[99]

The composers discussed in Part 1.3 usually work within quarter-tone through twelfth-tone systems. In his *Sonata for Solo Cello,* David Gorton uses *scordatura* tuning to the seventh partial, tablature notation and quarter-tone accidentals. The seventh partial provides access to patterns in 36TET and the quartertones to 24TET. The piece's pitch range is thus conceived within 72-tone temperament.

97 Ibid., 87.

98 Ben Johnston, String Quartet n.4 (unpublished, 1978).

99 Strange and Strange, *The Contemporary Violin: Extended Performance Techniques*, 156.

Ferneyhough's *Exordium* for string quartet uses quarter-tone and attached-arrows accidentals. In his performance notes he states 'Quarter tones are indicated by special symbols and are generally to be understood as tempered pitches. Intermediate pitches are preceded by symbols employing arrow-heads to show direction of pitch modification. The intonation of such pitches is comparatively more flexible, and is to be understood in part context sensitive.'[100] The arrow accidentals are thus adjusted in terms of the performer's perception of relative consonance and dissonance.[101]

2.3. Other

In contrast with Ferneyhough's example where the players will intuitively recur to JI ratios, Bartók and Cage have used arrow-heads to break proportions between pitches. In his String Quartet n.6 (Figure 2.14), Bartók uses the arrow sign to indicate (in theory) the lowering of a quarter-tone, but what he is looking for is the 'out-of-tune' effect of the microtonal clashes preceded by the rough sound of the *forte* down bows (as the movement's title, Burletta, might suggest). Bartók is not tuning his music to any specific system. Cage's *Freeman Etudes* (Figure 2.13) have arrow accidentals indicating indeterminate microtonal inflections. Each performer will (and is supposed to) interpret the deviation from the written pitch in a different way. Cage is after is the singularity and independence of each sound. There should be no intelligible relationship between the pitches (they were 'chosen' through chance operations[102]).

Fig 2.13. Cage, *Freeman Etude n.1.*[103]

[100] Ferneyhough, *Exordium* (Peters Edition, 2008)

[101] Ferneyhough doesn't leave the performer in any doubt of their authority over tuning, unlike Simon Holt's example or Artusi's double-sided position.

[102] Strange and Strange, *The Contemporary Violin: Extended Performance Techniques*, 66.

[103] John Cage, *Freeman Etudes,* Book 1 and 2 (Henmar Press, 1981).

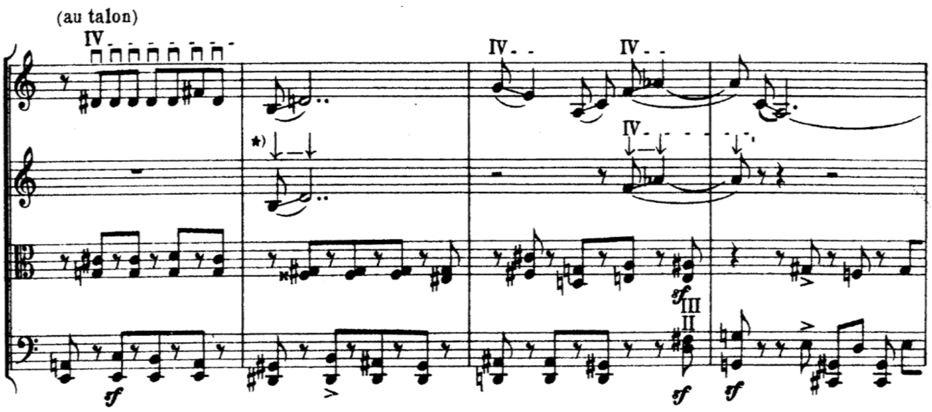

Fig 2.14. Bartók's String Quartet n.6, 3rd Mvt. Burletta, mm. 7-10.[104]

Giacinto Scelsi's String Quartet n.4 uses quarter-tone notation and is structured around beating harmonies of very narrow pitch clusters. The timbre of the piece is also affected by the *accordatura* tuning.

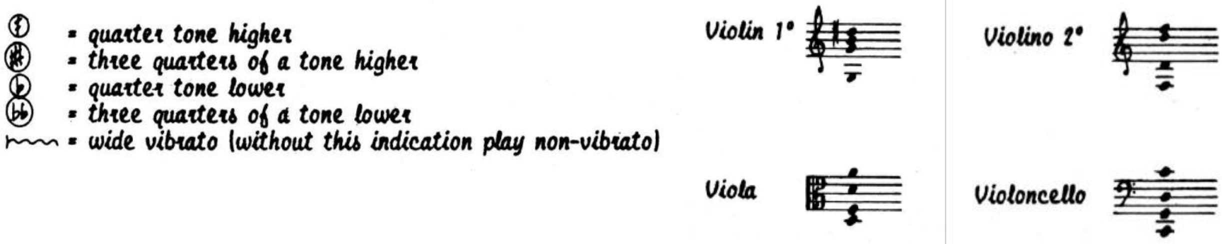

Fig 2.15. Scelsi's performance instructions for his String Quartet n. 4. [105]

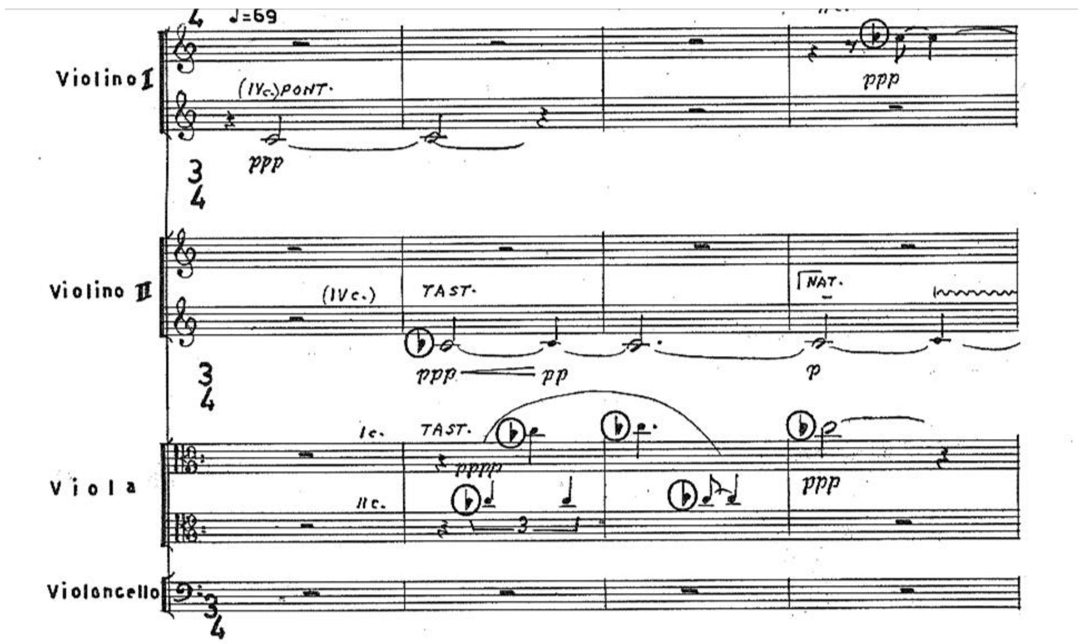

Fig 2.16. Opening of Scelsi's String Quartet n. 4.[106]

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104 Bela Bartók, String Quartet n.6 (Boosey & Hawkes, 1941).

105 Giacinto Scelsi, String Quartet n.4 (Ed. Salabert, 1983).

Alex Hills' *Chromatic Sedition* for two violins features the use of microtones in a 'harmonic, excessive vibrato, deviation, beating, quartertone, glissando' manner (in his own terms). The symbol in bar 59 indicates a wide vibrato. The pitch deviation of bar 61 should result in a beating of 3Hz, and thus in a quaver rhythm at ♪=180, coinciding with the previous tempo mark of ♩=90 in bar 45.

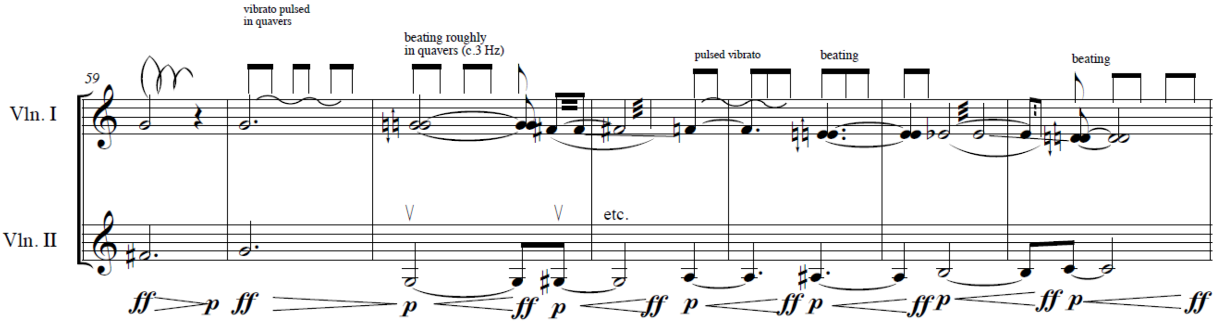

Fig 2.17. Alex Hill's *Chromatic Sedition*, mm. 59-65.[107]

Lastly, *scordatura* tuning provides practical access to a variety of tunings. It may be notated in tablature or at sounding pitch. *Fosdyke Wash* for piano quintet by David Gorton contains examples of both notations. Violin I is tuned with the A-string a third tone sharper and the G-string a third tone flatter. Violin II has the D-string a third tone flatter and the E-string a third tone sharper.

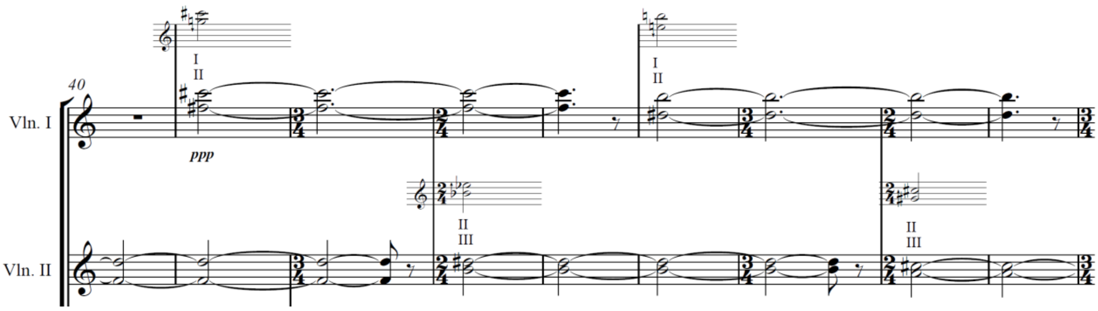

Fig 2.18. Gorton's *Fodyske Wash*, mm. 40-48.[108]

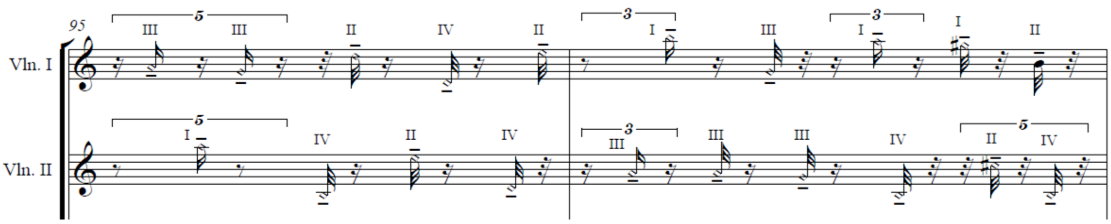

Fig 2.19. Gorton's *Fodyske Wash,* mm.95-96. [109]

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106 Ibid.

107 Hills, *Chromatic Sedition* (unpublished, 2014).

108 Gorton, Piano Quintet: *Fodyske Wash* (David Gorton Music, 2010).

Part 3. The impracticalities of 12-TET subdivisions

The use of the equally tempered quarter-tone in string music demands a performer can naturally play intuitively (in tune) in 12TET in order to subdivide its semitone into two equal steps, which does not actually happen in string music practice (or in singing as Artusi pointed out). Mieczyslaw Kolinski explains in "A New Equidistant 12-Tone Temperament"[110] that performers make intuitive adjustments depending on the 'musical function' of a pitch in relation to its melodic and harmonic context (the slight sharpening of leading tones that relates to Pythagorean tuning is a common example of this).

In *Psychology of Music,*[111] Carl Seashore analyzes recordings by Menuhin, Szigeti, Busch, Elman and other great violinist and shows statistic tables of the frequency and direction of pitch deviation from the 12TET scale: 'The violinist deviates over 60% of the time from the tempered scale notes with deviations 0.05 tone [10 Cents] or greater and over 31% of the time with deviations of 0.1 [20 Cents] tone or greater. The average deviation is of 0.1 tone.' These are the average data when comparing recordings of solo works with works with piano accompaniment. He continues to classify the deviations by intervals: '(a) the fourth degree [or subdominant] of the scale to be lowered [40% of the time] and the seventh [leading tone] raised [85% of the time], (b) minor and diminished intervals to be contracted [51%], (c) major and augmented intervals to be expanded [44%] and (d) chromatically altered notes to over-shoot the alteration in the direction of the chromatic used [50%].'[112] He concludes: 'the violinist, when unaccompanied, does not play consistently in either the tempered or the natural scale, but tends on the whole to conform with the Pythagorean scale in the intervals here tested.'[113] [114]

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109 Ibid.

110 Mieczyslaw Kolinski, "A New Equidistant 12-Tone Temperament", *Journal of the American Musicological Society*, Vol. 12, No. 2/3 (Summer-Autumn, 1959), 210-214.

111 Carl Seashore, *Psychology of Music* (New York, 1938).

112 Ibid., 212.

113 Ibid., 224.

114 Columns 1 and 5 of *Figure 3.2* show a cycle of fifths above C in sharps and below C in flats (reading Column 5 from end to top). Their corresponding Columns 3 and 7, show the just tuning in Cents and one can see that directions of the intonation deviations described by Seashore correspond to those from the series above C for notes that are sharpened, and below C notes that are flattened.

3.1. Theory

A Pythagorean scale derived from a row of 12 fifths distributes the pitches almost evenly within an octave. When tempering out this scale so that the 12th fifth and the generating tone coincide then each fifth is tempered by 1/12 of the Pythagorean comma and becomes the complex number[115] 2^(7/12) instead of 3/2, which the musical ear does not 'understand' in terms of degree of consonance (and cannot be visually calculated on a monochord). If a scale is derived from a cycle of 24 fifths, then the excess is doubled and the resolution to approximate 5-limit JI ratios is not improved. 24TET does not provide better approximations of just intervals encompassed within a 12TET scale, but it approximates the 11th harmonic with the 'in between' tones, giving the tempered quarter-tone.

My table (Figure 3.2) compares a cycle of 24 pure fifths and a closed circle of 24 tempered fifths. The fifth to eighth column are the scale calculated with descending fifths (equivalent to ascending fourths) from C and should be read from end to top. Column 1 and 5 give the note names in sharps and flats respectively which then become equivalent enharmonics in Column 10 where I have substituted the in-between tones of 12TET with quarter-tone accidentals (see Figure 3.1). As one can see comparing Columns 3 and 4, and 7 and 8, the in-between tones are very far apart from their JI relative fifths. However, Column 10 shows the 5-limit JI ratios that correspond to 12TET and for the quarter-tones one can see that they are a very accurate representation of the 11th harmonic. For example, on an open C-string the 11th partial will fall between a perfect fourth and a tritone giving the interval of C𝄳 – F, 16/11 or 648.7 Cents, which is only 1.3 Cents from the tempered quarter-tone. Other ratios involve the 11th as a multiple of the fraction or its denominator.

Sharp	Flat	Double-sharp	Double-flat	Quarter- sharp	Quarter-flat	Three-quarter sharp	Three-quarter flat
♯	♭	x	𝄫	𝄲	𝄳	𝄰	𝄳♭

Fig 3.1. Quarter-tone notation for Figure 3.2.

115 If 2 represents the octave, then 2^(n/n) is 2, and *n* times 2^(1/n)· … · 2^(1/n) is 2^(1/n+1/n … [*n* times]), therefore the fifth in 12TET (which is represented by the interval of 7 semitones) is 2^(7·1/12)= 2^(7/12).

1. Note Name	2. 3^p/2^q	3. Cents of 24 just fifths	4. 24 tempered fifths in Cents	5. Note Name	6. 2^p/3^q	7. Cents of 24 inverted just fifths	8. 24 tempered fifths in Cents	9. Enharmonic note names	10. Close JI harmonics and their Cents
C	$3^0/2^0$	0.00	0	A♭♭♭♭	$2^{39}/3^{24}$	1153.08	0	C, B#, D𝄫	1/1
G	$3^1/2^1$	701.96	700	E♭♭♭♭	$2^{37}/3^{23}$	655.03	700	G, Fx, A𝄫	3/2 (701.9)
D	$3^2/2^3$	203.91	200	B♭♭♭♭	$2^{35}/3^{22}$	156.99	200	D, Cx, E𝄫	9/8 (203.9)
A	$3^3/2^4$	905.87	900	F♭♭♭	$2^{34}/3^{21}$	858.94	900	A, Gx, B𝄫	5/3 (884.4)
E	$3^4/2^6$	407.82	400	C♭♭♭	$2^{32}/3^{20}$	360.90	400	E, Dx, F♭	5/4 (386.3)
B	$3^5/2^7$	1109.78	1100	G♭♭♭	$2^{31}/3^{19}$	1062.85	1100	B, Ax, C♭	15/8 (1088.3)
F#	$3^6/2^9$	611.73	600	D♭♭♭	$2^{29}/3^{18}$	564.81	600	F#, G♭	7/5 (582.5)
C#	$3^7/2^{11}$	113.69	100	E♭♭♭	$2^{27}/3^{17}$	66.76	100	C#, D♭	16/15 (111.7)
G#	$3^8/2^{12}$	815.64	800	B♭♭♭	$2^{26}/3^{16}$	768.72	800	G#, A♭	8/5 (813.7)
D#	$3^9/2^{14}$	317.60	300	F♭♭	$2^{24}/3^{15}$	270.67	300	D#, E♭	6/5 (315.6)
A#	$3^{10}/2^{15}$	1019.55	1000	C♭♭	$2^{23}/3^{14}$	972.63	1000	A#, B♭	16/9 (996.1)
E#	$3^{11}/2^{17}$	521.51	500	G♭♭	$2^{21}/3^{13}$	474.58	500	E#, F	4/3 (498)
B#	$3^{12}/2^{19}$	23.46	0	D♭♭	$2^{20}/3^{12}$	1176.54	0	C, B#, D𝄫	1
Fx	$3^{13}/2^{20}$	725.42	750	A♭♭	$2^{18}/3^{11}$	678.49	750	G‡ , A⫛	77/50 (747.5)
Cx	$3^{14}/2^{22}$	227.37	250	E♭♭	$2^{16}/3^{10}$	180.45	250	D ‡, E ⫛	64/55 (262.4)
Gx	$3^{15}/2^{23}$	929.33	950	B♭♭	$2^{15}/3^9$	882.40	950	A‡ ,B⫛	96/55 (964.3)
Dx	$3^{16}/2^{25}$	431.28	450	F♭	$2^{13}/3^8$	384.36	450	E‡, F𝄳	100/77 (452.5)
Ax	$3^{17}/2^{26}$	1133.24	1150	C♭	$2^{12}/3^7$	1086.31	1150	B‡ , C𝄳	11/6 (1049.4)
Ex	$3^{18}/2^{28}$	635.19	650	G♭	$2^{10}/3^6$	588.27	650	F𝄰, G 𝄳	16/11 (648.7)
Bx	$3^{19}/2^{30}$	137.15	150	D♭	$2^8/3^5$	90.22	150	C 𝄰, D 𝄳	12/11 (150.6)
Fx#	$3^{20}/2^{31}$	839.10	850	A♭	$2^7/3^4$	792.18	850	G𝄰 A 𝄳	18/11 (852.6)
Cx#	$3^{21}/2^{33}$	297.75	350	E♭	$2^5/3^3$	294.13	350	D𝄰 E 𝄳	11/9 (347.4)
Gx#	$3^{22}/2^{34}$	999.71	1050	B♭	$2^4/3^2$	996.09	1050	A𝄰 B 𝄳	11/6 (1049.4)
Dx#	$3^{23}/2^{36}$	501.66	550	F	$2^2/3^1$	498.04	550	F‡ G⫛	11/8 (551.3)
Ax#	$3^{24}/2^{38}$	3.62	0	C	$2^0/3^0$	0.00	0	C	1/1

Fig 3.2. Comparative table of 24-JI fifths cycle and 24TET.[116]

One can already imagine that music written in 24TET is both physically and psychologically challenging for the string player. Before proceeding to the challenge of 24TET, let's take a look at some of these psychological problems inborn already in 5-limit classical music.

Although the equally tempered piano would have been disseminated by Tchaikovsky's time, I argue that an analysis of Figures 3.3, 3.4 and 3.5 suggests his 'inner ear' had a Pythagorean approach to intonation and that his choice of spelling was not only a matter of correctness. If the Fx in was to be played in 5-limit JI, as it would have been in the 17th Century, the violin's range would not reach the note as it would be a Syntonic comma below the open G. I am not suggesting that the Fx should be played on a stopped G-string but that he internally heard the Fx sharper than the G. In a passage as such, there would be no need to finger an Fx since

116 Data from Column 10 is extracted from Partch, *Genesis of a Music,* 461-463.

violinists usually play loudly and with high velocity and weight on the bow, which (especially on an open string) slightly (but noticeably) raises the pitch.

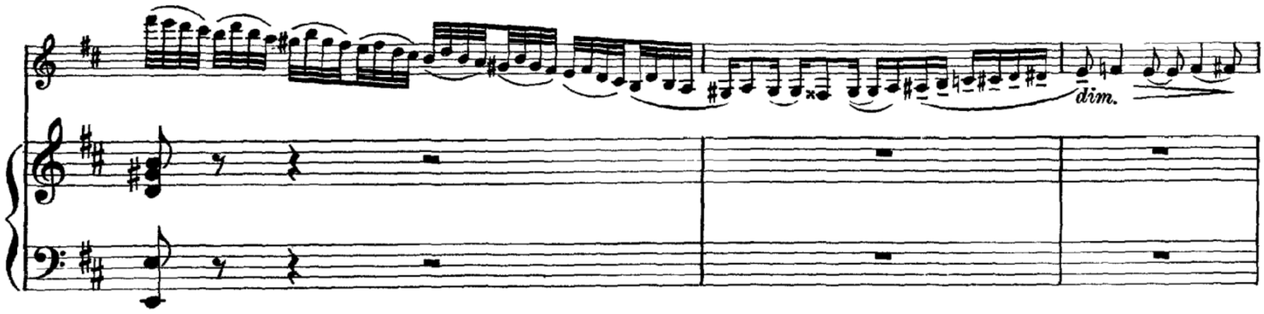

Fig 3.3. Tchaikovsky's Violin Concerto, 1st Mvt., mm. 66-68.[117]

Further examples of intonational ambiguity are abundant in progressions with augmented sixth chords. Regener argues that the interval ratio most suited to representing the augmented sixth in a lattice is the 7/4, which very closely approximates its corresponding interval in 31TET and this might have been how Vicentino understood it within his 31-tone model. [118] [119] However this 7/4-approach to the augmented sixth is not used in the performance of classical music (evidence shows Tartini's contemporaries did not approve of 7 Identity[120]).

Figures 3.4 and 3.5 show two different perspectives in augmented sixth spellings. In Mozart's example the V^7 in D♭ works as an augmented sixth in C major after the pause. If he had notated an F# instead of a G♭ the performers would have had to re-adjust the F# in order to get the V^7 'in tune' (and theoretically it would not have been a V^7). In contrast, Tchaikovsky's example contains a chromatic progression where the sixth is spelled as a sixth, although the soloist will understand it as a C until the very last moment when it leads to C# in a Plagal cadence in A major. In real time performance an augmented sixth does not truly become one until it has passed and resolved to the dominant; it is thus commonly tuned as a V^7 despite its possible different spellings. Again, I defend that Tchaikovsky's choice of spelling is a result of what Patrizio Barbieri describes as the 'pull of the tonic' resulting in 'Pythagorean-expressive' or 'functional intonation' which had already been noticed in the middle of

117 Tchaikovsky, Violin Concerto in D Major (G. Schirmer, 1918).

118 Eric Regener, "The Number Seven in the Theory of Intonation", *Journal of Music Theory*, Vol. 19, No. 1 (Spring, 1975), 140-153.

119 Partch, *Genesis of A music*, 377.

120 Ibid., 388.

the 18th Century by violinists such as Abbé Roussier or Bemetzreider.[121]

Fig 3.4. Mozart's String Quartet n.22 in B-flat major, 3rd mvt. *Menuet*, K589.[122]

Fig 3.5. Tchaikovsky's Violin concerto in D major, 1st mvt., mm. 119-129.[123]

121 Barberi and Mangsen, "Violin intonation: a historical survey", 82.

122 Mozart, String Quartet n.22 K589 (Breitkopf & Härtel, 1882).

In the following section, I will address the problems presented by 24TET from a (my) performer's perspective through the practice of three quarter-tone studies written by a student composer from the Royal Academy of Music.

3.2. Case-study I: *Three Quarter-tone Studies*

I first worked with quarter-tone material with composer Thanakarn Schofield. He presented two short studies that on the one hand have experimental value for him as a composer, and on the other served me to get into the sound world and timbre of the quarter-tone. Schofield was interested in exploring this musical idiom in the context he might present quarter-tones in a forthcoming larger work targeted at performers and audiences who are not familiar with this interval. At the same time the studies should also serve as an introductory exercise to the quarter-tone.

During the exploration of these studies we started a conversation on the aesthetics of the quarter-tone with questions such as: should an A# be higher than a B♭? And following that question, should an A♯ be higher than a B𝄳? This context is the genesis of the system with no equivalent enharmonics that I will present in Part 4.

The first two studies (Figures 3.7 and 3.8) only use four note-names (D, E, F and G) and are preceded by ten microtonal versions of the tetrachord (Figure 3.9) inspired by the Ancient Greeks' enharmonic genera, in order to prepare the performer for thinking in unusual intervallic patterns.

Fig 3.6. Schofield's performance instructions for the quarter-tone accidentals indicated in Cents.

Fig 3.7. Schofiled's *Quarter-tone Study n.1.*

[123] Tchaikovsky, Violin Concerto in D Major (G. Schirmer, 1918).

Fig 3.8. Schofield's *Quarter-tone Study n.2.*

Fig 3.9. Schofield's exercise on different tetrachords.

During my violin practice I noticed that two different methods were effective when learning to tune tetrachords: by treating quarter-tones as leading tones of half-tones and as equal subdivisions of a larger interval. Figure 3.10 shows my practice process. For example, the first pattern that I practised for the second tetrachord is based on an intuitive approximation of the F‡ , placing it as a leading note to F#. A second approach for this tetrachord is by dividing the space between E and G into two equal steps of 150 Cents (or more intuitively dividing the fingerboard space between these notes into approximately two equal steps) in the style of Ton de Leeuw's String Quartet n.2 (see Figure 2.6).[124]

In the same way I developed on two sketches for studies n.1 and n.2 in order to aid myself in the learning process (Figures 3.11, 3.12, 3.13, 3.14). In the edited versions (which could potentially serve for the pedagogy of microtonal performance practice) the notes with the downward tail are not part of the study, but rather indicate the interval of which the quarter-tone is the midpoint. The preceding or the following note dictates the range of this interval.

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124 For the third tetrachord I used my last approach as a starting point in order to calculate the distance from F ‡ to G# (of 250 Cents). When approaching the fifth tetrachord I first inserted the quarter-tones between the usual degrees of the chromatic scale chromatically, E𝄳 thus becoming an expressive appoggiatura of D, E‡ a leading tone to F, F‡ a leading tone to F# and so on. I marked those tones from the original tetrachord so I could visually internalise the pattern. The following three bars combine my two approaches. The E‡ is placed between the D# and F#, finally obtaining the first two intervals of the tetrachord by dividing equally the minor thirds and the last interval from the third tetrachord by finger memory. In the eigth tetrachord I played a scale with an F 9/8 above E♭, so that I would find the G𝄳 as its (imaginary) appoggiatura. In the other tetrachords I applied the same method.

Fig 3.10. My practice sketch for Schofield's tetrachord exercise.

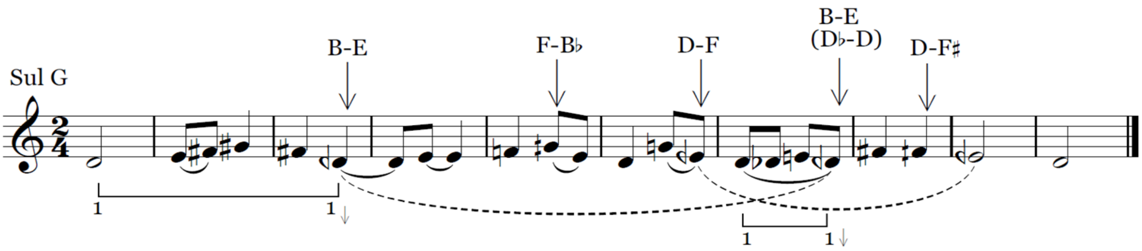

Fig 3.11. My practice notes on *Quarter-tone Study n.1*. The line below the staff indicates the lowering of the D, and the dashed lines indicate common tones to help visual memory.

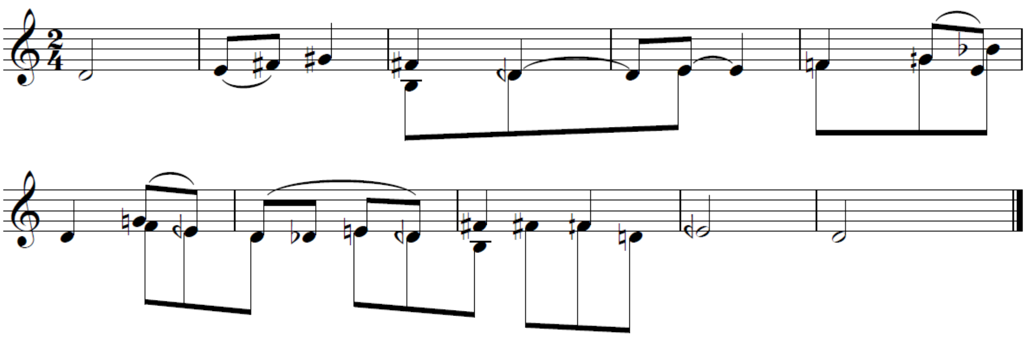

Fig 3.12. Developed version of my practice method.

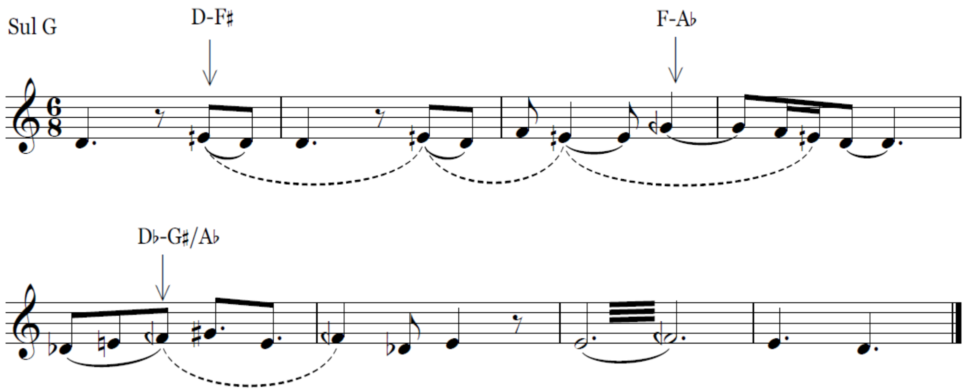

Fig 3.13. Practice notes on *Quarter-tone Study n. 2.*

Fig 3.14. Developed version of my practice process for *Quarter-tone Study n.2.*

A third study came out of empirically trying 9TET scale patterns on an open-string drone. 9TET sounds somewhat Middle Eastern and can be notated easily. It is constructed using tempered minor thirds with an equal division within each third throughout the octave. The 9TET scale can be played on a violin with little effort but this study includes quarter-tone double-stops which defy JI and Pythagorean tuning that violinists favour. This is why this study is less beautiful than the previous monodic studies. The harmonies become rapidly complex when writing music in quarter-tones and it is at this point one starts considering the harmonic possibilities a rational scale (JI system) instead of an intervallic scale (ETs).

Fig 3.15. Schofield's *Quarter-tone Study n. 3.*

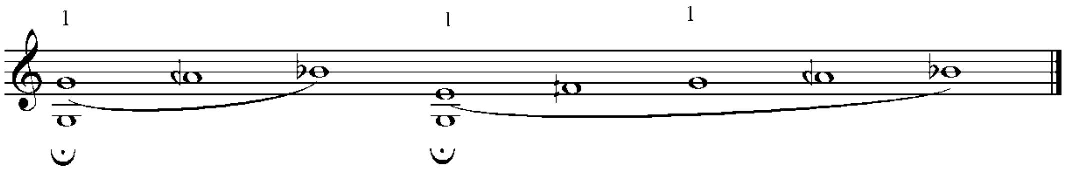

Fig 3.16. Practice sketch for *Quarter-tone Study n. 3.*

Learning these studies led to the two most important questions that generated most of the material that will be presented in the following sections. Schofield and me agreed that the absence of the diatonic and chromatic semitone in the piano was detrimental for musical expression and that the quarter-tone should be treated like the diatonic and chromatic semitones on a smaller scale. But *where* and *how* was I supposed to place the quarter-tone within a semitone? We also noted that if dealing with double-stops a better system was to be explored in the field of JI tunings in order to (aurally) understand the relationship between two or more pitches.

Part 4. A practical approach to 53TET

In this chapter I will present a 53 tone scale in which the smallest step is a tempered Pythagorean comma. This is the result of tempering a cycle of 53 fifths, where the 53rd pure fifth has an excess of 3.62 Cents from the generating tone. This had already been noticed in China by the 1st Century B.C. by Ching Fang.[125] [126] To temper out and close the cycle each fifth is modified by 0.68 Cents (3.62/53). In Europe, Nicolau Ramarinus built a 53TET keyboard in 1640.[127] In the second half of the 19th Century 53TET keyboards were developed by James Paul White and Bosanquet.[128] They valued this scale because it represents intervals derived through the 3rd and 5th harmonic with a deviation of only 0.1 to 1.5 Cents (this deviation can be seen comparing Columns 8 and 10 from Figure 4.3). [129] A comparison of a chromatic scale of semitones and 53rd-tones to the octave can be visualized in Figure 4.2. The 53TET scale deviates much less from its untempered version than the 12TET.

Microtonal composers have spoken about the effect that intonation has on timbre.[130] [131] As has been shown in Part 3, contemporary musicians have developed a predilection for Pythagorean tuning in violin music. This intonational sensitivity is also referred to as the violinist's concern with 'beauty of tone' rather than with 'theories of harmonies or the scale of New Music [equal temperament]' by Ll. S. Lloyd.[132]

But how does this 'beauty of tone' that Lloyd talks about reveal itself in physical terms? Figure 4.1

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125 Ernest G. McClain and Ming Shui Hung, "Chinese Cyclic Tunings in Late Antiquity", *Ethnomusicology*, Vol. 23, No. 2 (University of Illinois Press, May 1979), 208.

126 Partch refers to Ching Fang as King Fang. Partch, *Genesis of A music*, 368.

127 Ibid., 438.

128 Ibid., 392.

129 Ibid., 434.

130 'I don't advocate tuning the piano in other ways other than twelve equal. Whereas harpsichords and organs work well in meantone, on a piano the meantone fifths have a terribly hard nasal sound, and the pure thirds don't seem to resonate right. Likewise, the violin sounds bad if you make any change to those pure perfect fifths. Particularly for single lines, the best tuning of the violin is the Pythagorean, which is what violinists prefer unless they are playing slow double-stops. As for the brass they sound very bad with out-of-tune fifths of thirds, especially in the high register' Easly Blackwood in Keilsar, "Six American Composers on Nonstandard Tunings", 185.

131 '[…] to get the intonation, the players might use alternate fingerings, lip the notes high and low, or pull out the barrel of the instrument, all of which change the timbre. You get a flute that sounds like a shakuhachi, or instead of an oboe you get different colors of oboe.' Ben Johnston in Keilsar, "Six American Composers on Nonstandard Tunings", 184-185.

132 Ll. S. Lloyd, *The Musical Ear* (Oxford University Press, 1940), 79.

shows an excerpt from one of Schofield's pieces. During the creation of this work, we discussed whether the C♮ at the beginning of the second line should be notated as a B# instead.[133] It could be thought either as a leading-tone between two C#s (in which case the last C from the first line should also be a B#) or a major second below D, as indicated by the upward note-tales following the voice C-B-D-C from the end of the first line. However, I noticed that a B# resonated much better than a C. The violin resonates very well on notes derived through the ascending cycle of fifths which leads Seashore to note that the violinists' intonational mistakes are 'preponderantly in the direction of sharpening.'[134] Theoretically undertones don't exist[135] which means that a C, for example, can't be related to a G unless the G is regarded as its second overtone (overtone series on G does not contain C). This means that a C on a violin does not provoke the sympathetic resonance of the violin strings' overtones. However, the B# is part of the ascending cycle of fifths and also lies nearer the 10th overtone (or 11th partial, C𝄳 of the G string than C. This resonance can be strongly felt and is favoured by the violinist, which explains my preference for the suggested change.[136]

The value of the 53TET scale in the context of my project is that it presents a number of microtones that are easily accessible for string players, and within the bound of a reasonable level of approximation (see Tovey's quote below), can be accommodated by the modern violinist without psychological strain: 'The subject of just intonation is fatally fascinating to people whose mathematical insight has not attained to the notion of approximation. In art, as in mathematics, accuracy lies in the establishing the relevant degree of approximation rather than in unrolling interminable decimals.'[137]

Fig 4.1. Excerpt from *Sonata Anathema,* II. Passion, for solo violin by Schofield.[138]

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133 I argued that it should be changed to B# while he believed it should be retained as a C.

134 Seashore, *Psychology of Music,* 212.

135 In practice it is possible to play C below the G-string by manipulating the pressure and velocity of the bow. Mari Kimura is re-knowned for her refined 'subharmonics' technique. "The world below G and beyond" (CD, Mutable Music, 2010).

136 The notation change was not approved by Schofield, but we agreed that I would play a B# on that particular notated C!

137 Sir Donald Tovey, *Encyclopedia Britannica*, 14th ed. (art. "Harmony"). Ll. S. Lloyd, *The Musical Ear*, 1.

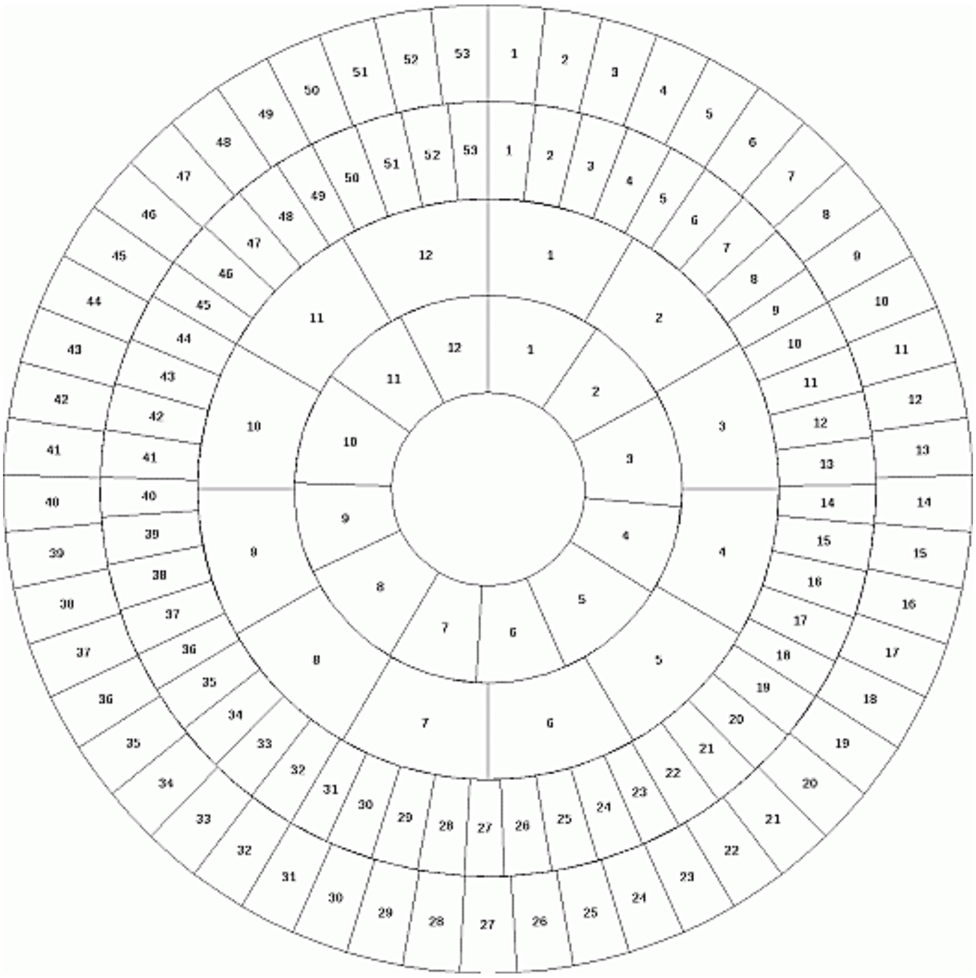

Fig 4.2. Comparative graph of 12 pythagorean semitones, 12 tempered semitones, 53 pythagorean 53rd-tones and 53 tempered 53rd-tones to the octave.

4.1. Theory

53TET not only allows the cycle of fifths to be closed notably more accurately than 12TET, but also contains the chromatic and diatonic semitone differentiated by the Pythagorean comma that is so commonly used in expressive intonation, involving and enlargement of the chromatic semitones (5 times 1200/53, i.e., 113 Cents approx.) and a compression of the diatonic semitones (4 times 1200/53, i.e., 91 Cents approx.) in comparison to the 12TET semitone (100 Cents). In 53TET the interval of 1200/53 Cents is the smallest step in the scale, which since it constitutes the difference between the two familiar semitones described above can be noticed easily by the string player.

138 Thanakarn Schofield, *Sonata Anathema II* (unpublished, 2013).

The following table (Figure 4.4) shows an ascending cycle of fifths (Column 1) and a descending cycle of fifths (Column 5, to be read from end to top) with its respective JI ratios (Columns 2 and 6) and Cents (Columns 3 and 7). Columns 4 and 8 show the equally tempered version of the fifths in Cents. Unlike Figure 3.2, the 53TET scale that will be exposed here has no enharmonics (with an exception on the E ‡ / F 𝄳 and B ‡ / C 𝄳 , which practically should be differentiated by approx. 10 Cents). Column 9 shows the note-names and alterations used in my 53TET scale which are explained in Figure 4.3.

Fig 4.3. Microtonal accidentals in my 53TET scale. Numbers indicate an approximation in Cents.

From this table (Figure 4.4) one can see that this notation is unusual because the pure third would be represented with the note names C – F♭ for example, and the pure major sixth would have to be written as C – B𝄫.

1. Note Name	2. 3^p/2^q	3. Cents of just fifths	4. Cents of tempered fifths	5. Note Name	6. 2^P/3^Q	7. Cents of (inverted) just fifths	8. Cents of tempered fifths	9. New Note Names	10. Close JI Harmonics and their Cents
C	$3^{0}/2^{0}$	0.00	0.00		$2^{85}/3^{53}$	1196.38	0.00	C	1/1
G	$3^{1}/2^{1}$	701.96	701.89		$2^{83}/3^{52}$	698.34	701.89	G	3/2 (702)
D	$3^{2}/2^{3}$	203.91	203.77		$2^{81}/3^{51}$	200.29	203.77	D	9/8 (203.9)
A	$3^{3}/2^{4}$	905.87	905.66		$2^{80}/3^{50}$	902.25	905.66	A	27/16 (905.9)
E	$3^{4}/2^{6}$	407.82	407.55		$2^{78}/3^{49}$	404.20	407.55	E	14/11 (417.5)
B	$3^{5}/2^{7}$	1109.78	1109.43		$2^{77}/3^{48}$	1106.16	1109.43	B	40/21 (1115.5)
F#	$3^{6}/2^{9}$	611.73	611.32		$2^{75}/3^{47}$	608.11	611.32	F#	10/7 (617.5)
C#	$3^{7}/2^{11}$	113.69	113.21		$2^{73}/3^{46}$	110.07	113.21	C#	16/15 (111.7)
G#	$3^{8}/2^{12}$	815.64	815.09		$2^{72}/3^{45}$	812.02	815.09	G#	8/5 (813.7)
D#	$3^{9}/2^{14}$	317.60	316.98		$2^{70}/3^{44}$	313.98	316.98	D#	6/5 (315.6)
A#	$3^{10}/2^{15}$	1019.55	1018.87		$2^{69}/3^{43}$	1015.93	1018.87	A#	9/5 (1017.6)
E#	$3^{11}/2^{17}$	521.51	520.75		$2^{67}/3^{42}$	517.89	520.75	E#	27/20 (519.5)
B#	$3^{12}/2^{19}$	23.46	22.64		$2^{65}/3^{41}$	19.84	22.64	B#	81/80 (21.5)
Fx	$3^{13}/2^{20}$	725.42	724.53		$2^{64}/3^{40}$	721.80	724.53	Fx	32/21 (729.2)
Cx	$3^{14}/2^{22}$	227.37	226.42		$2^{62}/3^{39}$	223.75	226.42	Cx	8/7 (231.2)
Gx	$3^{15}/2^{23}$	929.33	928.30		$2^{61}/3^{38}$	925.71	928.30	Gx	12/7 (933.1)
Dx	$3^{16}/2^{25}$	431.28	430.19		$2^{59}/3^{37}$	427.66	430.19	Dx	9/7 (435.1)
Ax	$3^{17}/2^{26}$	1133.24	1132.08		$2^{58}/3^{36}$	1129.62	1132.08	Ax	64/33 (1146.8)
Ex	$3^{18}/2^{28}$	635.19	633.96		$2^{56}/3^{35}$	631.57	633.96	F ♯	16/11 (648.7)
Bx	$3^{19}/2^{30}$	137.15	135.85		$2^{54}/3^{34}$	133.53	135.85	C ♯	12/11 (150.6)
Fx#	$3^{20}/2^{31}$	839.10	837.74		$2^{53}/3^{33}$	835.48	837.74	G ♯	18/11 (852.6)
Cx#	$3^{21}/2^{33}$	341.06	339.62		$2^{51}/3^{32}$	337.44	339.62	D ♯	11/9 (347.4)
Gx#	$3^{22}/2^{34}$	1043.01	1041.51		$2^{50}/3^{31}$	1039.39	1041.51	A ♯	20/11 (1035)
Dx#	$3^{23}/2^{36}$	544.97	543.40		$2^{48}/3^{30}$	541.35	543.40	F ‡	11/8 (551.3)
Ax#	$3^{24}/2^{38}$	46.92	45.28		$2^{46}/3^{29}$	43.30	45.28	C ‡	33/32 (53.2)
Ex#	$3^{25}/2^{39}$	748.88	747.17		$2^{45}/3^{28}$	745.26	747.17	G ‡	32/21 (729.2)
Bx#	$3^{26}/2^{41}$	250.83	249.06		$2^{43}/3^{27}$	247.21	249.06	D ‡	7/6 (266.9)
Fxx	$3^{27}/2^{42}$	952.79	950.94		$2^{42}/3^{26}$	949.17	950.94	A ‡	12/7 (933.1)
Cxx	$3^{28}/2^{44}$	454.74	452.83		$2^{40}/3^{25}$	451.12	452.83	E ‡. F 𝄳	9/7 (435.1)
Gxx	$3^{29}/2^{45}$	1156.70	1154.72	A♭♭♭♭	$2^{39}/3^{24}$	1153.08	1154.72	B ‡. C 𝄳	64/33 (1146.8)
Dxx	$3^{30}/2^{47}$	658.65	656.60	E♭♭♭♭	$2^{37}/3^{23}$	655.03	656.60	G 𝄳	16/11 (648.7)
Axx	$3^{31}/2^{49}$	160.61	158.49	B♭♭♭♭	$2^{35}/3^{22}$	156.99	158.49	D 𝄳	11/10 (165)
Exx	$3^{32}/2^{50}$	862.56	860.38	F♭♭♭	$2^{34}/3^{21}$	858.94	860.38	A 𝄳	18/11 (852.6)
Bxx	$3^{33}/2^{52}$	364.52	362.26	C♭♭♭	$2^{32}/3^{20}$	360.90	362.26	E 𝄳	11/9 (347.4)
Fxx#	$3^{34}/2^{53}$	1066.47	1064.15	G♭♭♭	$2^{31}/3^{19}$	1062.85	1064.15	B 𝄳	11/6 (1049.4)
Cxx#	$3^{35}/2^{55}$	568.43	566.04	D♭♭♭	$2^{29}/3^{18}$	564.81	566.04	G ф	11/8 (551.3)
Gxx#	$3^{36}/2^{57}$	70.38	67.92	E♭♭♭	$2^{27}/3^{17}$	66.76	67.92	D ф	33/32 (53.2)
Dxx#	$3^{37}/2^{58}$	772.34	769.81	B♭♭♭	$2^{26}/3^{16}$	768.72	769.81	A ф	14/9 (764.9)
	$3^{38}/2^{60}$	274.29	271.70	F♭♭	$2^{24}/3^{15}$	270.67	271.70	F ф	7/6 (266.9)
	$3^{39}/2^{61}$	976.25	973.58	C♭♭	$2^{23}/3^{14}$	972.63	973.58	B ф	7/4 (968.8)
	$3^{40}/2^{63}$	478.20	475.47	G♭♭	$2^{21}/3^{13}$	474.58	475.47	G 𝄬	21/16 (470.8)
	$3^{41}/2^{64}$	1180.16	1177.36	D♭♭	$2^{20}/3^{12}$	1176.54	1177.36	D 𝄬	160/81 (1178.5)
	$3^{42}/2^{66}$	682.11	679.25	A♭♭	$2^{18}/3^{11}$	678.49	679.25	A 𝄬	40/27 (680.5)
	$3^{43}/2^{68}$	184.07	181.13	E♭♭	$2^{16}/3^{10}$	180.45	181.13	E 𝄬	10/9 (182.4)
	$3^{44}/2^{69}$	886.02	883.02	B♭♭	$2^{15}/3^{9}$	882.40	883.02	B 𝄬	5/3 (884.4)
	$3^{45}/2^{71}$	387.98	384.91	F♭	$2^{13}/3^{8}$	384.36	384.91	F♭	5/4 (386.3)
	$3^{46}/2^{72}$	1089.93	1086.79	C♭	$2^{12}/3^{7}$	1086.31	1086.79	C♭	15/8 (1088.3)
	$3^{47}/2^{74}$	591.89	588.68	G♭	$2^{10}/3^{6}$	588.27	588.68	G♭	7/5 (582.5)
	$3^{48}/2^{76}$	93.84	90.57	D♭	$2^{8}/3^{5}$	90.22	90.57	D♭	21/20 (84.5)
	$3^{49}/2^{77}$	795.80	792.45	A♭	$2^{7}/3^{4}$	792.18	792.45	A♭	11/7 (782.5)
	$3^{50}/2^{79}$	297.75	294.34	E♭	$2^{5}/3^{3}$	294.13	294.34	E♭	32/27 (294.1)
	$3^{51}/2^{80}$	999.71	996.23	B♭	$2^{4}/3^{2}$	996.09	996.23	B♭	16/9 (996.1)
	$3^{52}/2^{82}$	501.66	498.11	F	$2^{2}/3^{1}$	498.04	498.11	F	4/3 (498)
	$3^{53}/2^{84}$	3.62	1200.00	C	$2^{0}/3^{0}$	0.00	1200.00	C	1/1

Fig 4.4. Comparative table of the 53 cycle of fifths and 53TET.[139]

[139] Column 10 is extracted from Partch, *Genesis of A Music*, 430.

4.2. Cage and Feldman

In the defence of my systematic approach to (extended) Pythagoreanism as a microtonal system I would first like to refer to two pieces which demonstrate the origins of my perspective: *Cheap Imitation* by John Cage and *For John Cage* by Morton Feldman (although my work were not inspired by theirs).

In *Cheap Imitation*, Cage instructs the performer to use Pythagorean intonation. He gives an overview of how it works concluding:

> Hence the tone B♯♯ is higher than the Octave of C by the small interval $\frac{74}{73}$ (=24 cents), and the tone D♭♭ is lower than the Octave below C by the same interval. If we ascend by perfect Fifths from C to A𝄪, we shall find the same constant difference between
>
> C G D A E B F♯♯ C♯♯
> D♭♭ A♭♭ E♭♭ B♭♭ F♭ C♭ G♭ D♭
>
> G♯ D♯ A♯ E♯ B♯ F𝄪 C𝄪 G𝄪 A𝄪
> A♭ E♭ B♭ F C G D A B .
>
> The tones in the upper line are all higher than those in the lower by the small interval $\frac{74}{73}$ (=24 cents).

Fig 4.5. Excerpt from *Cheap Imitation*'s performance notes.[140]

The work was first written in 1969 for piano solo and in 1975 Paul Zukofsky comissioned this new version for solo violin. Cage was fascinated by Zukofsky's intonational skills which according to Marc Sabat were 'legendary' as he was able to 'flexibly articulate intervals in pure tunings as well as micro-subdivisions of the tempered scale.'[141] The introductory note by Cage states this: 'I study under Zufosky's patient the tutelage, not how to play the violin, but how to become more baffled by its almost unlimited flexibility. Cheap Imitation for solo violin is one of the results of this study.'

The piece is transposed a major third higher than the original *Cheap Imitations* for piano, therefore starting on a D# instead of a B. The violin version is particularly full of sharps and double sharps (Figure 4.6). Cage is after the expressive use of intonation and its possible shadings. In fact, he is not after a precise

140 John Cage, *Cheap Imitation* (Henmar Press, 1977).

141 "New Music Box", accessed 09/2014, http://www.newmusicbox.org/articles/Feldmans-Composition-for-violin-1984-Extended-Just-Intonation-in-interpretative-practice/.

interpretation of Pythagorean intervals: 'Rather than making this difference of 24 cents precise, the violinist should sometimes increase it, even to 40 cents, in order to distinguish tones from one another.'[142]

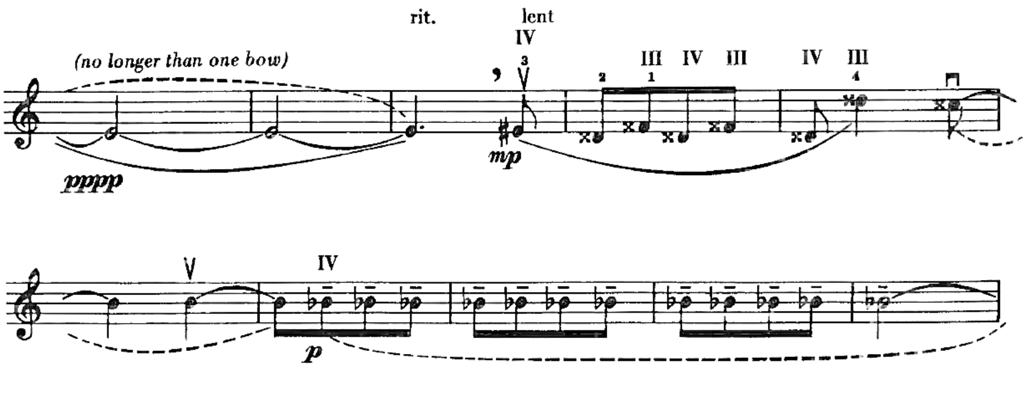

Fig 4.6. Excerpt from *Cheap Imitations* for solo violin.[143]

In 1982, Morton Feldman wrote *For John Cage* for Cage's 70th birthday celebration concert which was premièred by Paul Zukofsky and Aki Takahashi.[144] The piece uses an unconventional looking spelling of the accidentals and no performance notes are attached to the score (Figure 4.7). According to Marc Sabat, Zukofsky defended a mean-tone approach to this piece. We can only speculate on this matter, but assuming Morton would have been aware of Cage's enharomnic spelling in *Cheap Imitations* I would defend a Pythagorean approach.

My approach to Pythagorean intonation as a microtonal resource has the same principles as Cage's *Cheap Imitation* (if not Feldman's as well). However, mine differs from these examples since I attempt to systematize Pythagoreanism as a microtonal structure. My approach is particularly on display in the *53TET 2-Part Counterpoint* (Figure 4.19).

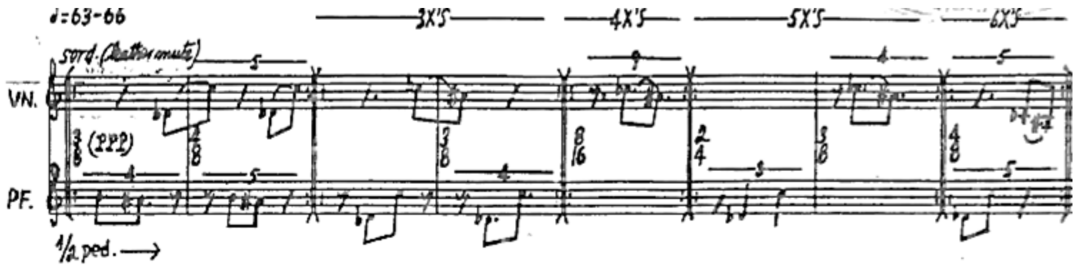

Fig 4.7. Opening of Feldman's *For John Cage.*[145]

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142 Cage, *Cheap Imitation* (Henmar Press, 1977).

143 Ibid.

144 "New Music Box", accessed 09/2014, http://www.newmusicbox.org/articles/Feldmans-Composition-for-violin-1984-Extended-Just-Intonation-in-interpretative-practice/.

145 Morton Feldman, *For John Cage* (Universal Edition 1982).

4.3. Notation for a 'new' 53TET scale

Figure 4.2 shows the microtonal accidentals of my scale. I have substituted those pitches expressed with more than a double-flat or double-sharp (shown in Figure 4.8) with a microtonal accidental (see Column 9 in Figure 4.3*)*. ‡ and ◁ can be considered small quarter-tone accidentals. The # and ♭ are chromatic semitone accidentals, i.e., larger half-tones. ♯ and ◁♭ are 2/3rd-tone accidentals. The double-sharps and double-flats can be thought of either as eight-tone accidentals of the proceeding upper pitch class of the scale, 9/8th–tone accidentals, or as leading tones of sharp or flat accidentals, depending on the musical context.

Figure 4.9 shows all the pitches within an octave ordered from high to low with my notation and the nearest Cent approximation with respect to the starting G.

Fig 4.8. Traditional notation for a cycle of fifths shown by Johsnton in *Maximum Clarity.*[146]

[146] Johnston, *Maximum Clarity and other writings on music*, 21.

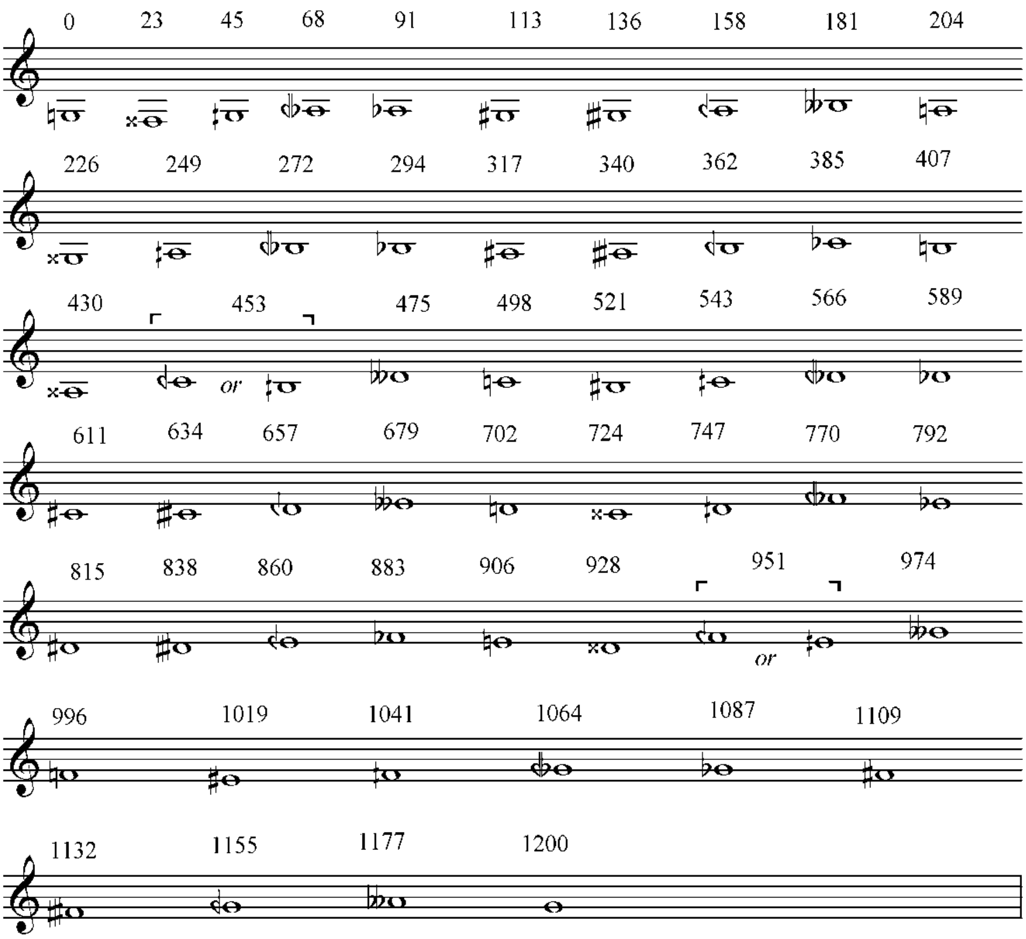

Fig 4.9. My 53TET scale from G to G'. Numbers indicate the deviation from G in Cents.

Fig 4.10. Johnston's 53-tone JI scale.[147]

147 Johnston, *Maximum Clarity and other writings on music*, 26.

In contrast with my scale, Johnston places the flats below the sharps (Figure 4.10) using the 25/24 chromatic semitone and the 27/25 diatonic semitone that would have been favoured by pre-18th-century musicians. Apart from the less common approach to sharps and flats from the perspective of modern violin playing (although performers following the Historically Informed Performance movement will be very familiar with this), Marc Sabat draws attention to another impractical implication of his asymmetrical system: the fifths notated as B♭-F, D-A, F#-C# should sound smaller than the pure fifth by one Syntonic comma.[148]

In order to simplify the notation for the 53 tempered fifths into a unique but logical chromatic scale pattern I made some arbitrary decisions (illustrated in Figures 4.11 and 4.12):

1. The notation of the accidentals would be restricted to those available from Sibelius'6.

2. The pitches and their alterations needed to maintain certain proportions between them, mirroring the Pythagorean division of the major second: G-A♭-G#-A or 1/1-256/243-2187/2048-9/8.

3. Therefore, the resulting chromatic ascending scale (using only sharps) and the downward (using only flats) mirror each other's step patterns.

4. Having the # and x the same distance a part (four 1/53rd–tones) from the unaltered tone and from its sharpened version respectively. And that this proportion would be mirrored by their relative flats and the quarter-tone and double-third tone signs (Figure 4.13).

5. Another decision I had to make was whether I wanted to keep the pitch increased by approx. 70 Cents as an upper third tone or as a lower double-third tone of the following note. I chose the double-third tone lowering and sharpening since they are heard more easily and would complement the 4th decision above.

An interval-name classification is something that has been conceived as a tool to aid familiarization with unusual temperaments, as for example in the Microtonal Trumpet Project.[149] However, as more notes are used, naming becomes more complex, and the invention and mastery of a 53TET interval name classification is beyond the scope of this project. Figure 4.14 shows all the harmonic intervals possible in 53TET the sound of which will be discovered in the *53TET 2-Part Counterpoint* in the final section of this chapter.

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148 "New Music Box", accessed 09/2014, http://www.newmusicbox.org/articles/intonation-and-microtonality/.

149 "The Microtonal Trumpet", accessed 09/2014 http://www.microtonaltrumpet.com.

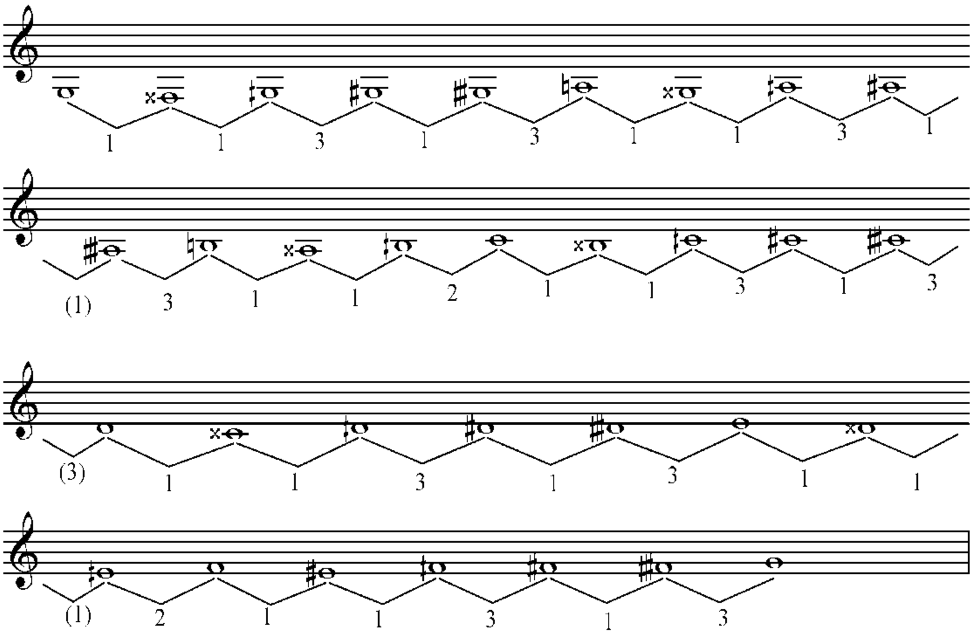

Fig 4.11. Ascending 53TET scale. Numbers indicate 1/53rd-tone steps.

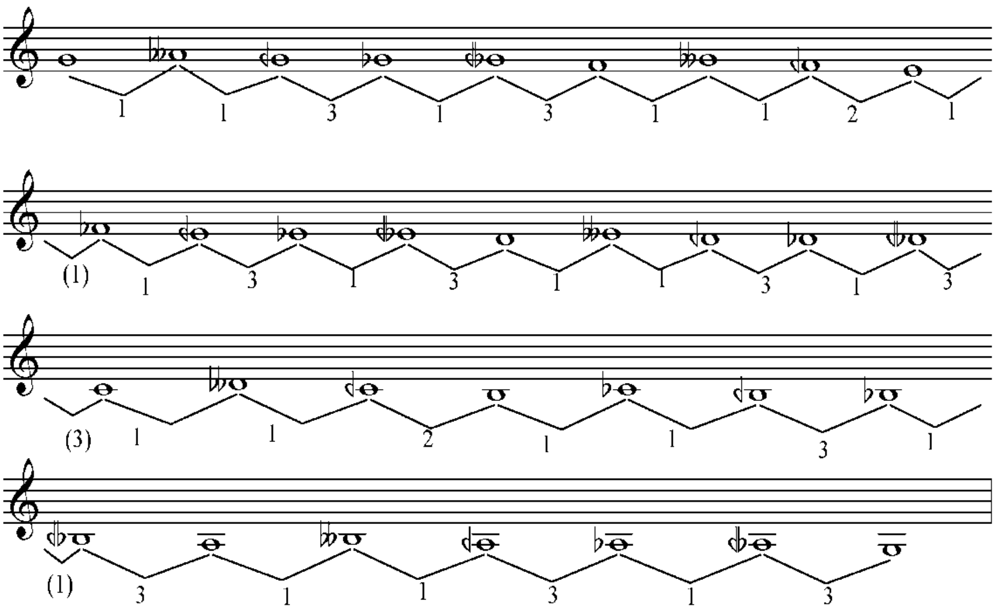

Fig 4.12. Descending 53TET scale.

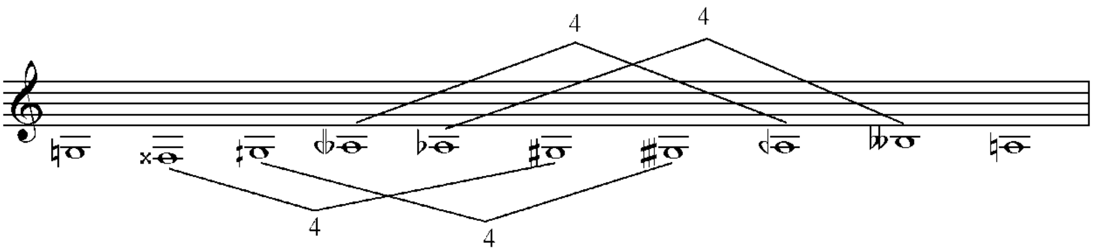

Fig 4.13. The indicated notes are 4/53rd-tones apart

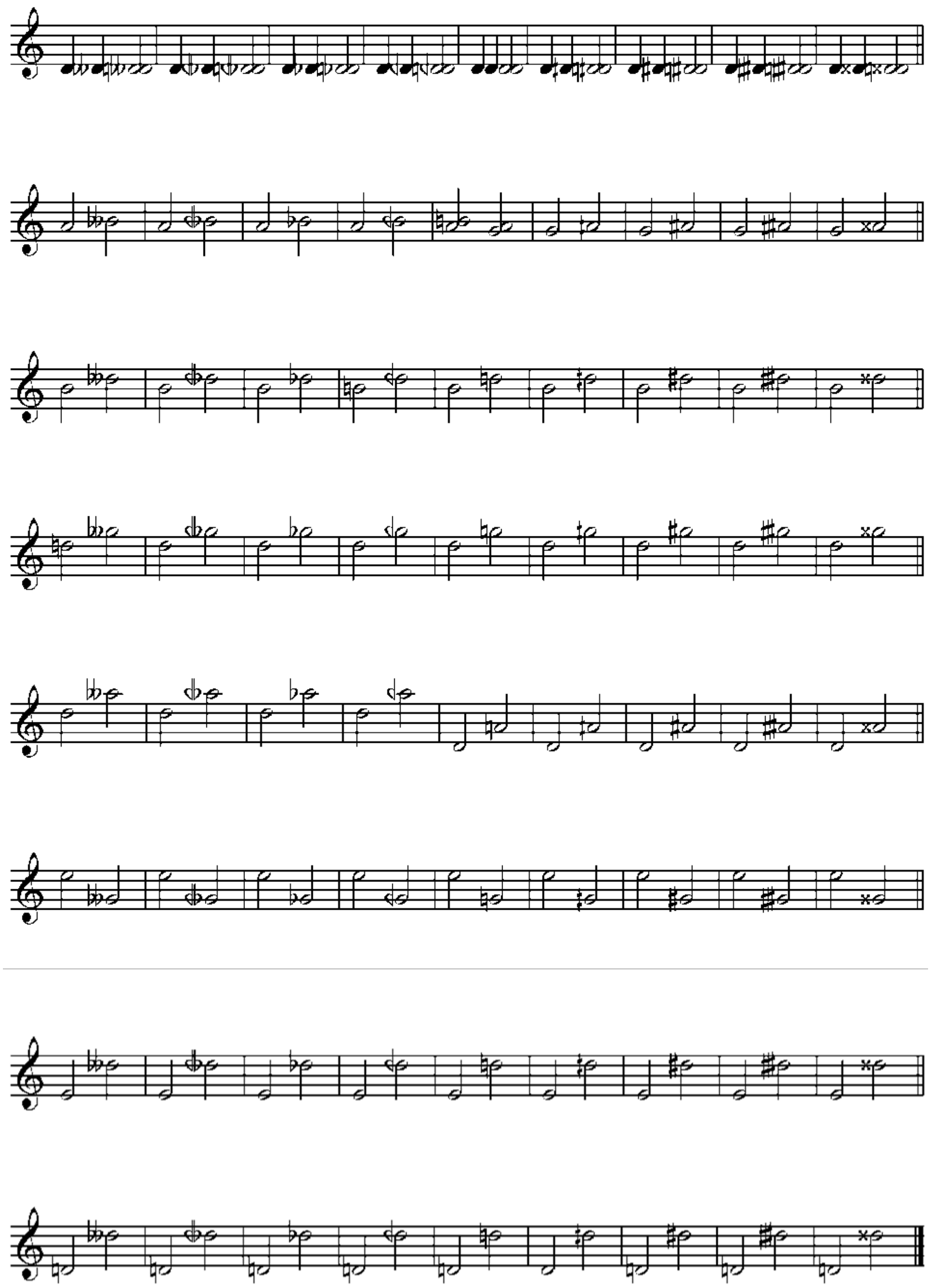

Fig 4.14. 72 different possible 'interval-spellings' in 53TET within an octave.

4.4. Case-study II: *53TET Study, 53TET Fugue* and *53TET 2-Part Counterpoint*

I wrote an introductory study to present the richness of 53TET. In the opening, 'expressive' intonation should be unavoidable, since the melody moves by 4ths and 5ths. As the study progresses, the performer is challenged to intonate the different diatonic and chromatic semitones in the same manner required in Cage's *Cheap Imitation*. Bars 23 to 24 present a drastic change in 'colour' (or spectra) given by the contrast of the G from the ascending fifths cycle and the surrounding flats from the descending cycle.

Fig 4.15. ***53TET Study.***

I developed two warm-up exercises. The first exercise (*Figure 4.16*) plays with the idea of adding and subtracting intervals of either four or nine 53rd steps. No new microtonal notation is involved. *Exercise n.2* introduces the interval of a 5/53 step and the more unusual 7/53 step. First, a pattern is introduced. The bracketed lines indicate the interval of a Pythagorean comma (-1 and +1), and the dashed line indicates a common tone, to which the B# is the leading tone. The scheme was formulated to provide a visual aid for the performer. The pattern consists of a repeated a semitone above and proceeds through 4 different transpositions introducing the quarter-tone, three-quarter tone and double-third tone accidentals. That way, the performer starts to tune intervals by ear (aural memory from the first pattern on C) and in expressive intonation as well as starting to internalize the function of the 'new' accidentals.

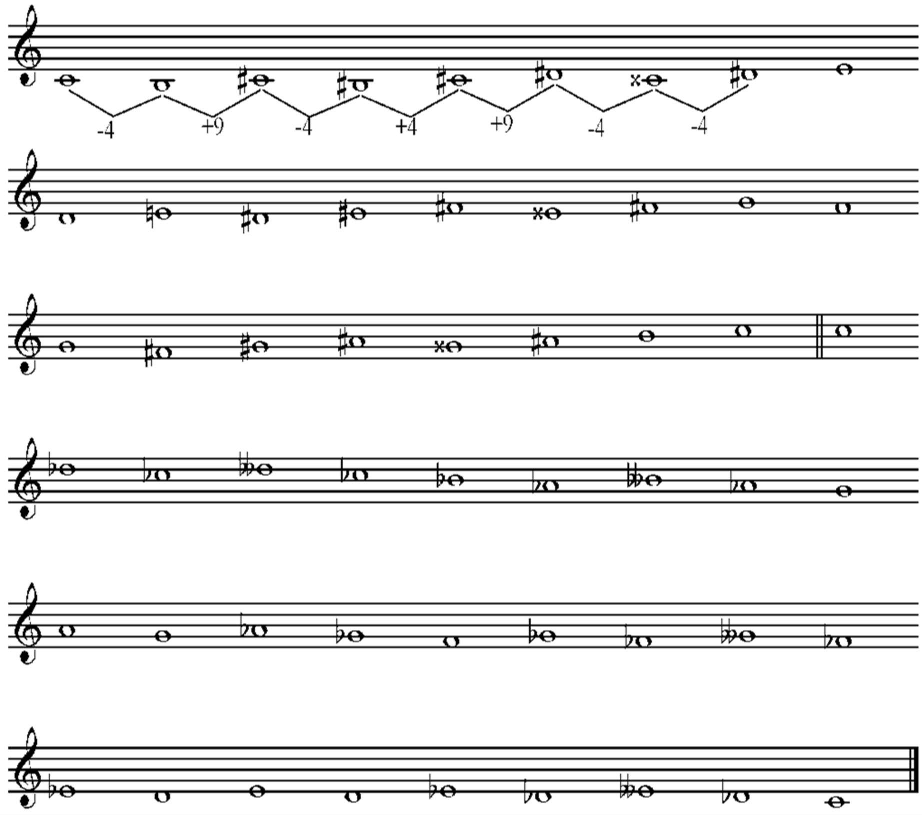

Fig 4.16. ***53TET Exercise n.1.***

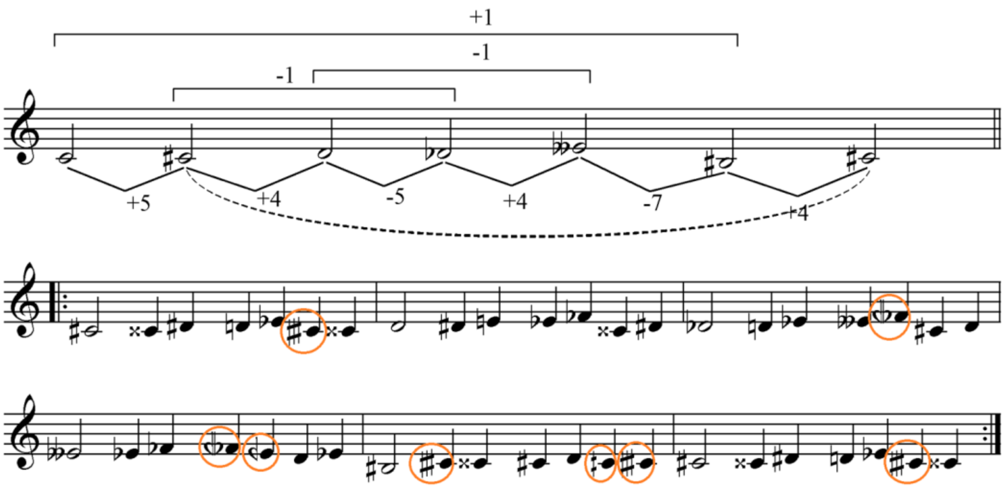

Fig 4.17*. 53TET Execrise n.2.*

The *53TET Fugue* confirms the value of this added level of ‘colour’ in the pitch range with its microtones working as leading tones of leading tones. It also adds a level of complexity for the performer, since all microtonal accidentals from my scale are included. It is inspired by the Fugue in Bartók’s Sonata for Solo Violin.

Fig 4.18. ***53TET Fugue.***

Figure 4.19 shows another study-piece I wrote during the London/Nashville Exchange Project (explained further in Part 6) inspired by the architecture of Renaissance species counterpoint. It introduces harmonic intervals in pairs, from the unison to the perfect fifth within a 53TET framework. This is probably my most innovative approach to Pythagoreanism as a microtonal resource.

The work is to be played by two string groups (or it could also be played by one performer on each part). Performers are required to play with *very* expressive intonation, without trying to adjust to the other group's note (i.e., avoiding 5-limit JI versions of the same intervals). Group 1 uses only sharp alterations whereas Group 2 uses only flat alterations. The numbers in the boxes indicate the number of 53rd-tone steps.

In the workshop where this study was first tested, the players understood the instructions very quickly and a new and dramatic sound world emerged during the first reading. The intervals were not respected strictly at all times and performers occasionally adjusted their intonation towards simple JI because of the surprise generated by unusual harmonies. Microtones are not used simply to add colour, but they present a new harmonic language that embraces the comma as a musical and harmonic interval (as in LaMonte Young's *The Well-Tuned Piano*).

Fig 4.19. *53TET 2-Part Counterpoint* study piece.

Part 5. *Seven Microtonal Variations* on a plainchant

While I was working on the explorations presented in Parts 3 and 4, I decided to collaborate with a composer to create a new piece that would showcase some of the microtonal approaches that I have discussed, that could be of use for performers who wish to experience microtonal music and for composers who desires to explore the possibilities of this language for solo violin. The final version of the piece contains seven variations that embrace various microtonal 'techniques' such as *scordatura*, *glissandi*, *molto vibrato* as an extended technique, JI ratios (accessed through natural harmonics and triadic harmonies), quarter-tones (an enharmonic system and a non-enharmonic one), third-tones, sixth-tones, 53rd-tones, and the way in which tuning practices can be used as a shock effect. At the same time the piece presents different approaches to notation and, together with the techniques used, it makes reference to works discussed in this project such as Xenakis' *Mikka* pieces (Figure 2.5) in *Variation III* and Gorton's *Fodyske Wash* (Figures 2.18 and 2.19) in the choice of *scordatura* and tablature notation.

Again, I collaborated with Schofield. We had four practical sessions together during which we brought our ideas and sketches and experimented with them. At first, I suggested techniques and presented my performer ideas so that he could respond to these ideas with sketches. However, the composer-role progressively became evenly distributed between the two of us and we ended up spending part of our time discussing which variations each of us should write. The practical sessions became a place to put into practice what we had already sketched. Some new ideas came out during moments of performer-composer exchange, such as the techniques used for the first variation and the choice of *scordatura* tuning. In our collaboration the composer's space was not defined and I ended up making the final decisions on order, transition sections and editing (always under the composer's approval).

5.1. Plainchant

Just as Schofield had been inspired by enharmonic tetrachords from the Ancient Greeks in his *Three Quarter-tone Studies,* the microtonal variations are based on a Gregorian Chant. Singing a chant demands the same aural awareness as playing a solo melody (without using the open strings): the only pitch of reference is the previously played or sung.

Johnston outlines the similarity between the singing practice of plainchant and the way a performer hears and thinks when playing twelve-tone serialism[150] (which may not necessarily be true when using fixed pitch instruments for atonal music). This is because in serialism there are no tonal relationships, which encourages intervallic thinking that is prevalent in *a capella* singing or unaccompanied playing. In contrast with the practice of singing Gregorian Chant, Eastern Indian monody is sung always together with a fixed pitch drone[151] and their musical intervals can be described using simple integer ratios. *Quarter-tone study n.3* uses an open string drone but the melody is constructed intervalically (as in *Quarter-tone studies n.1* and *n.2*), the absence of JI ratios results in the harsh beating sound between some tones.

The use of the *diesis* from the enharmonic genera in Guido's monochord divisions is still present.[152] In "Evidence for the Traditional View of the Transmission of Gregorian Chant",[153] David Hughes presents examples of possible instances where microtonal inflections would have been sung in a plainchant. He compares Gregorian chant manuscripts from various regions classifying the differences in notation in order to explain how these variants may affect our understanding of the transmission of the chants. One of the variants considered by Hughes is the different representations of the pairs E-F, B-C and A-B♭ in different manuscripts of the same chant. Most of the variants shown change the note E to F, B to C or A to B♭.[154] The noticeable differences suggest microtonal inflections might have taken place at these junctions and that notation had yet to be standardized. These examples coincide with Guido's division for the *diesis*.

The theme used in the *Seven Microtonal Variations* is one of the chants analyzed by Hughes. Figure 5.1 shows Hughes' research on the different settings of passages that include a semitone step or possibly a quarter-tone step. Figure 5.2 shows the manuscript and Figure 5.3 shows Prof. Jeremy Summerly's transcription.

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150 Johnston, *Maximum Clarity and other writings on music*, 134.

151 Ibid., 133.

152 Guido D'Arezzo, *Micrologus.*

153 David G. Hughes, "Evidence for the Traditional View of the Transmission of Gregorian Chant", *Journal of the American Musicological Society*, Vol. 40, No. 3 (University of California Press 1987), 377-404.

154 'The second reading in this column [referring to a different chant than *Figure 5.1*] and the variant in column m are both of the E-F type and involve only late Eastern sources. The picture may suggest a tradition of singing E high (E-F).' Ibid., 388.

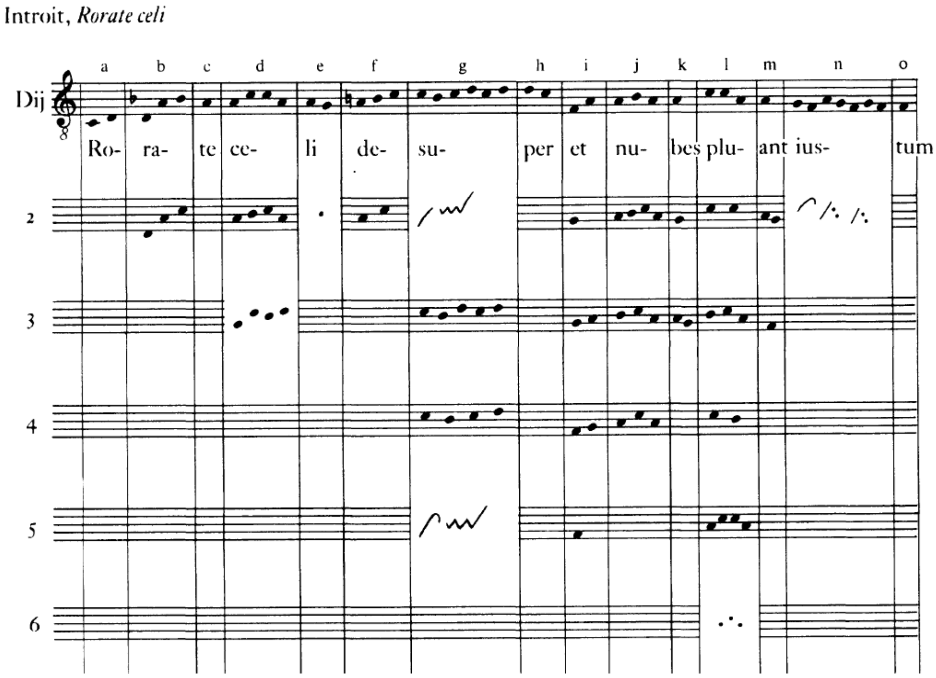

Fig 5.1. Different variants on *Rorate celi* shown by Hughes.[155]

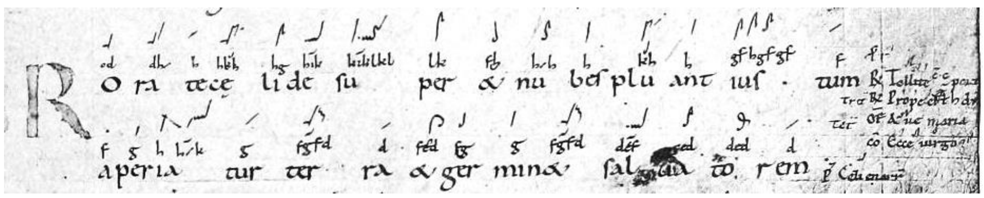

Fig 5.2. Manuscript.[156]

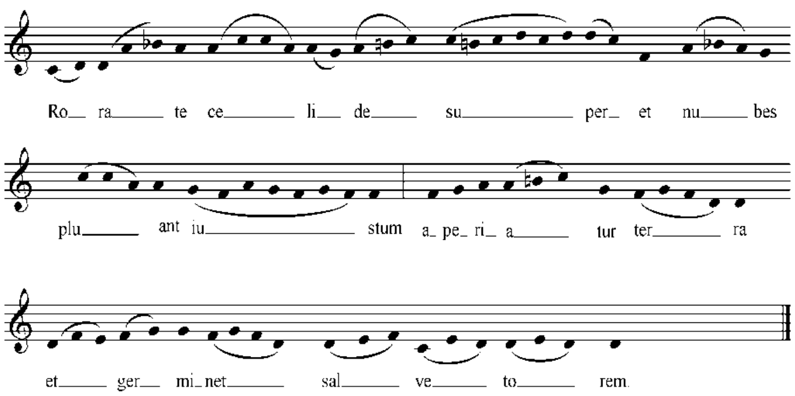

Fig 5.3. Transcription by Prof. Jeremy Summerly.[157]

155 Hughes, “Evidence for the Traditional View of the Transmission of Gregorian Chant”, 393.

156 Paleographie Musicale, *Les Principaux Manuscripts de Chant Gregorien, Ambrosien, Mozarabe, Gallician*, Vol. 51, *Introit. Protus authenticus: Rorate celi* (Solesmes, 1901), 13ff.

157 Jeremy Summerly, personal message (London, Feb. 2014)

5.2. Case-study III: the variations[158]

Scordatura tuning plays a key structural role in the variations. The A and E string are tuned to A=424Hz, and the D and G strings to A=442Hz. This offers 3 groups of string-pairs tuned: a fifth apart at A=442Hz (G and D), a fifth plus a third-tone apart at A=424Hz (D and A) and a fifth apart at A=424Hz (A and E). The presence of *scordatura* is strictly hidden in some of the variations which only use one string or one pair of consecutive strings tuned in fifths.

Since the *scordatura* requires a pair of strings to be tuned a third-tone lower than usual, the tablature pitch patterns are aligned with the seventh harmonic (7/4, which is a sixth tone lower than the interval of a minor seventh represented by the ratio 16/9) of the other strings tuned at A=442Hz. The seventh harmonic of the strings tuned a third-tone lower is also aligned to the tuning of the other pair of strings (Figure 5.4).

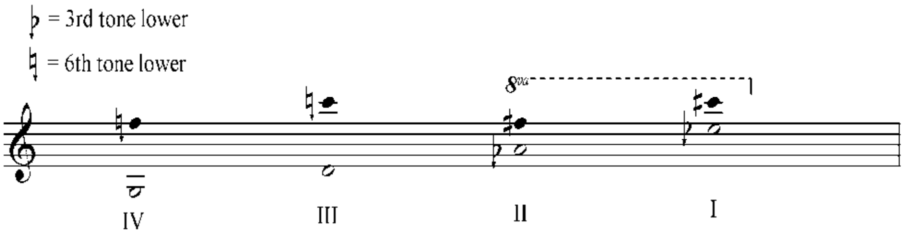

Fig 5.4. Pitches with black note-heads indicate the sounding pitch of the 7th harmonic of the open strings indicated with white note-heads, at A=442Hz.

The authorship of *Variations I, II* and *VI* belongs to Thanakarn and that of Variations *III, IV, V* and *VII* to myself, although a great deal of exchange took place between us throughout the creative process.

The theme is presented in *Variation I*. It features what for us, as Western classical musicians, would usually be considered an excessive use of very wide vibrato. However, such vibrato is not unusual in some non-classical genres, such as *erhu* playing in Chinese popular music. Vibrato is a common technique that one is often instructed to eliminate when performing microtonal music as in Xenakis' *Phlerga* in which he indicates 'Play no vibrato at all', or in Ligeti's *scordatura* passage in his Violin Concerto demanding *non. vibrato* to 'avoid deviations in intonation'[159]. This puts vibrato in the category of microtonal extended techniques.[160] This variation is to be

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158 Please refer to the score attached at the end of this chapter.

159 Ben Johnston also states that 'typical vibrato is too large for some of these intervals, so you have to decide what kind of vibrato use

played on the G-string (therefore at A=442Hz) and it includes some quarter-tones that remove any tonal feeling given by the B♭ and E that could have suggested C major and F major tonalities.

Schofield originally wrote *Variation II* for the violin's strings to be tuned in fifths but after some experimentation we realized the powerful effect created by the sudden change of reference tuning and decided to keep it as in tablature notation in *scordatura* tuning.[161] The following variation is written in the style of Xenakis' *Mikka* pieces (Figure 2.5), imitating both his notation and technique, i.e. using only sharp quarter-tones, which should be played in strict 24TET. In contrast, *Variation IV* presents different enharmonic spellings of quarter-tone intervals, suggesting that quarter-tones work as expressive leading-tones, inflections or passing notes around the melody.

Partials constitute the 'discrete' nature of the violin's sound. *Variation V* showcases different groupings of harmonics that fall into the same reference tuning pitch, as shown in Figure 5.4, and was written upon trial-error basis, thus my performer-role played an essential part in the writing of this variation. Both sounding pitch and tablature notation are shown.

Schofield wrote *Variation VI,* which is supposed to be played without *scordatura*. In order to re-tune the strings in fifths, I wrote a transition section where the tuning of the D and G strings to A=424Hz is included as a musical *glissando.* I presented my 53TET notation to Schofield and how it relates to expressive intonation, and, after studying my graphics he wrote this very challenging variation. Several notation adjustments had to be made before it could be recorded.

The final variation emulates historically informed performance style and, again, demands that all strings be tuned to A=424Hz. The performer is instructed to play in JI. Pitch deviations by a Syntonic comma are indicated by a + or –. Unlike Johnston, my scale of reference is a Pythagorean C Major diatonic scale.[162]

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and how much, if any.' Keilsar, "Six American Composers on Nonstandard Tunings", 186.

160 The idea of using vibrato as an extended technique here came from Schofield, who showed me a video of an *erhu* player and asked me to imitate his technique during one of the sessions. He then suggested to use a wooden mute, which together with the vibrato effect made it sound very unlike a violin.

161 This juxtaposition of different tunings has been used by Gloria Coates, although she makes use of them simultaneously rather than consecutively, in her *Lyric Suite for Piano Trio*, where the violin is tuned down a quarter-tone.

162 Marc Sabat has re-notated some of Bach's solo works in the same manner in *Sei Bach-Intonazioni per Violino Solo*. "Marc Sabat", accessed 09/2014, http://www.marcsabat.com/pdfs/worklist.pdf.

Seven Microtonal Variations

Scordatura tuning:
E and A strings tuned to A = 424 Hz.
D and G strings tuned to A = 442 Hz.

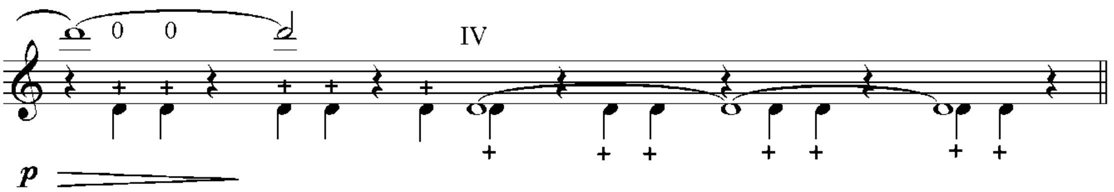
By Thanakarn Schofield and Sara Cubarsi
London 2014

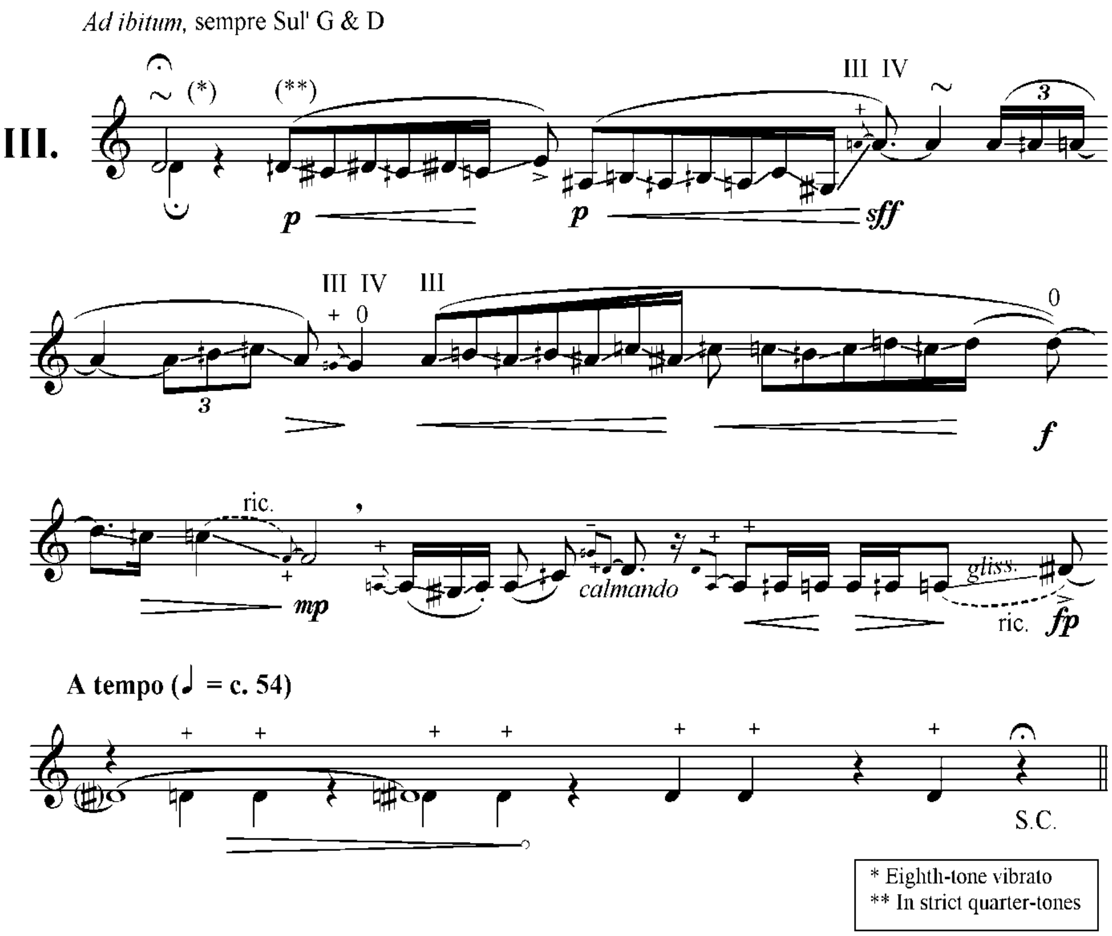
Like improvising, ♩ = c. 66

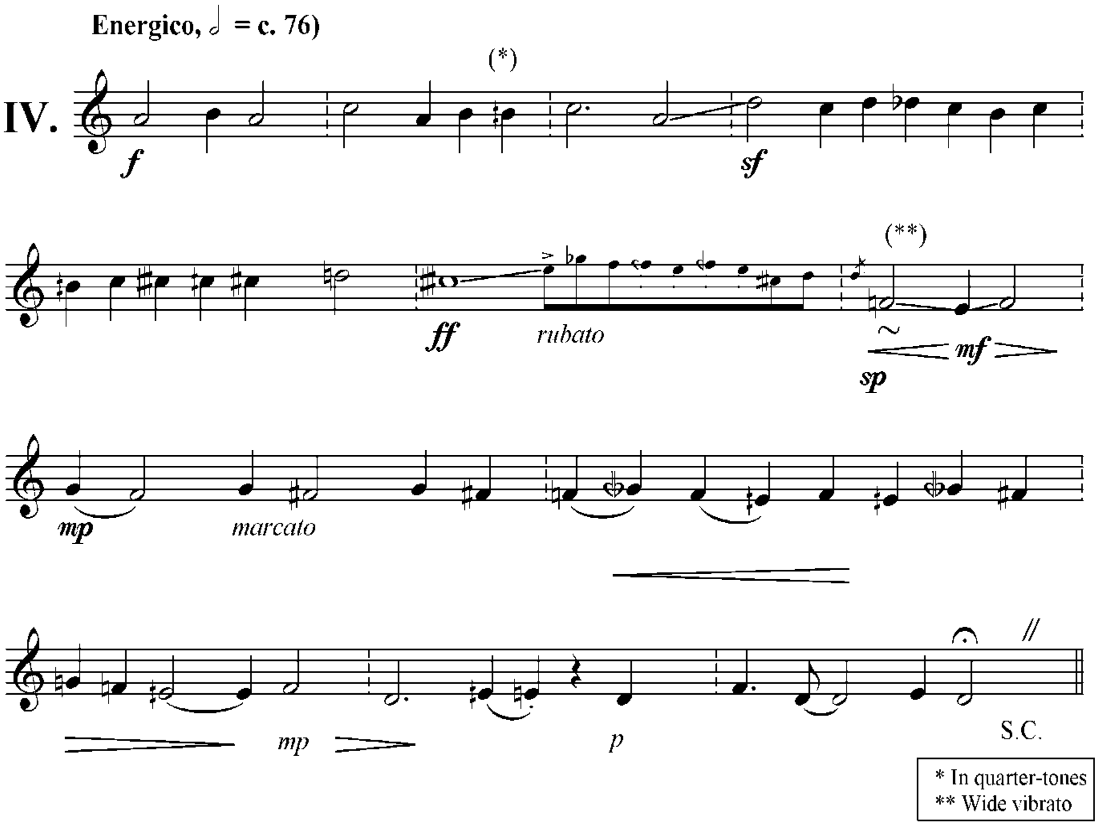
* Imitating the sound of the Erhu. Ancient Chinese violin

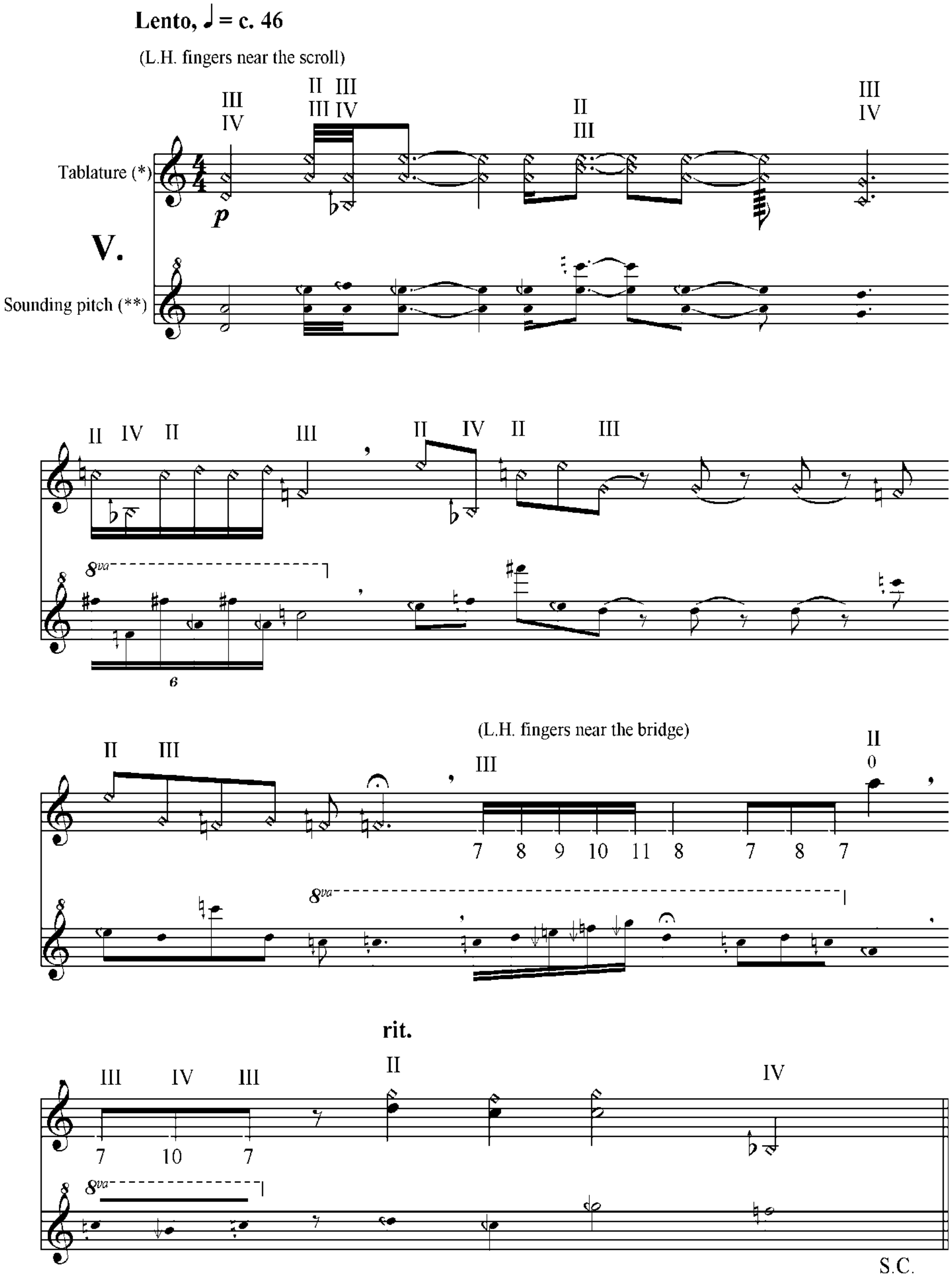
♩ = 54

*Approximate quarter-tone inflections

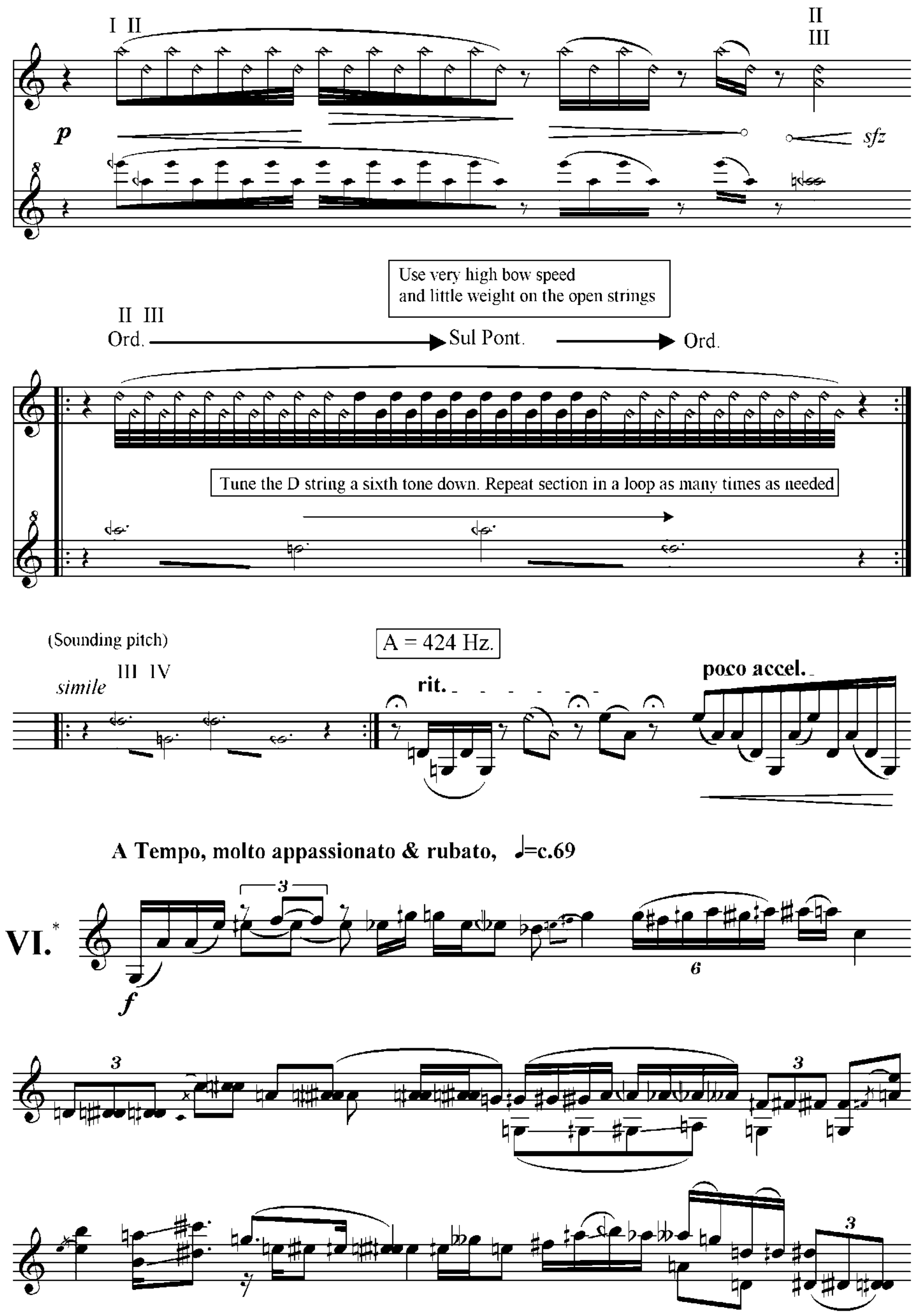



Ad ibitum, sempre Sul' G & D
III.
(*)
(**)
p
p
sff
III IV
III IV
III
0
f
ric.
mp
calmando
gliss.
ric.
fp
A tempo (♩ = c. 54)
S.C.
* Eighth-tone vibrato
** In strict quarter-tones

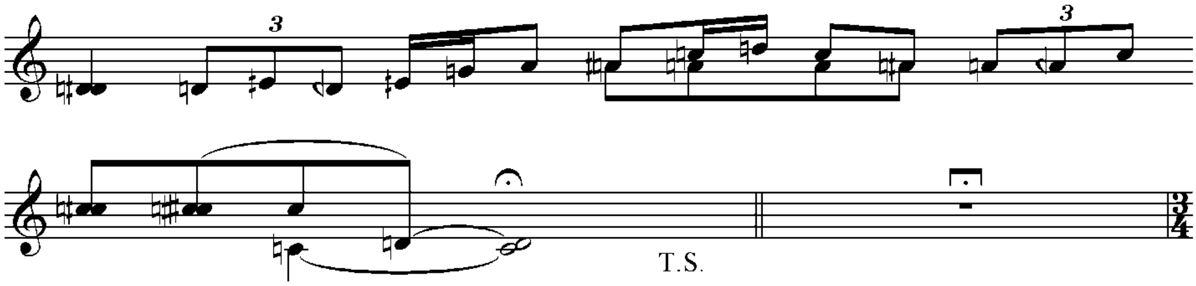

Energico, 𝅗𝅥 = c. 76)
IV.
(*)
f
sf
ff
rubato
(**)
sp
mf
mp
marcato
mp
p
S.C.
* In quarter-tones
** Wide vibrato



* Arrows attached to the accidentals indicate the placing of the pitch in tablature about a sixth tone upwards
** Arrows attached to the accidentals indicate the pitch sounds a sixth tone lower.The quarter-tone accidental indicates the pitch sounds a 1/3-tone lower. Arrows on their own indicate the pitch sounds slighty lower.



I II
II
III
p
sfz
Use very high bow speed
and little weight on the open strings
II III
Ord.
Sul Pont.
Ord.
Tune the D string a sixth tone down. Repeat section in a loop as many times as needed
(Sounding pitch)
A = 424 Hz.
simile
III IV
rit.
poco accel.
A Tempo, molto appassionato & rubato, ♩=c.69
VI.*
f
3
6



T.S.

* ‡/ᵹ = +/- 45 cents ♯/♭ = +/- 113c. ‡♯/♭♭ = +/- 136c. ×/♭♭ = +/- 226c.

Andante

In a Baroque style (*), con sord. leggero

VII.

S.C.

* The double stops are to be played in Historical Just Intonation. + amd - indicate raising or lowering the note by a Syntonic Comma with respect to a C Major scale in "Pythagorean-expressive" intonation. Freely ornamented.

Part 6. Case-study IV: contemplating the microtone

Last February I participated in the exchange project between the Blair School of Music (Vanterbilt University, Nashville) and the Royal Academy of Music. The aim of this project was to explore creative collaboration between performers and composers in a workshop environment. The team consisted of 4 composers, 5 violinists, 1 cellist and 1 double-bassist. During this two-week period I had a chance to develop and put into practice three experimental pieces. The first two, deal with the essence of the microtone, giving the microtone the smallest possible infinite value which is dictated by the individual performer and by their interaction with another performer. The third study was the *53TET 2-Part Counterpoint Study* presented in Part 4.19.

6.1. *Microtonal Spiral*

In this piece, I transform the sound *continuum* into a dense set of sounds by transforming the glissando technique into what I termed the 'discrete-glissando' technique. The arrhythmic texture, the high accumulation of clashing overtones and slow changing harmonies create (in Leibniz's words) a 'state where everything is worth of being observed, that is, the state of agreement or identity in variety; you can even say that [perfection] is a degree of contemplatibility.'[163]

The glissando-technique is achieved by the performers passing around a pitch and slightly raising or flattening it by a minimal frequency either to their player to their right or to their left (see Figure 6.1). The size of this frequency is dictated by each performer's aural skills. The glissando does not constitute a *continuous* but rather a discrete set of solid microtones which are piled up onto each other and distributed in different 'pitch-zones'.

This could also be performed by any large number of string instruments. I envisaged a set of 40 violins around the audience in a concert hall. However, the fact that we had a double-bassist in our team gave a level of depth to the sound spectrum that could not have been achieved in a violin ensemble. Even though this is a very

163 G. W. Leibniz, *Philosophical Essays* (Roger Ariew & Daniel Garber, Indianapolis, Hackett Publishing Company, 1989), 233-234.

dramatic study piece encapsulated by the opening B and its climactic tritone, this is about the exercise itself (contemplation) and the product should be *worthy of being observed* whatever the instrumentation and number of players.

THE MICROTONAL SPIRAL

For Double-bass, 5 Violins and Cello

– Sit in a circle.

– The VC opposite the DB.

– The DB starts with a B'': *forte*, tenuto, round sound and slow bow speed. The VN to its left plays the same note. The VN then raises it by the minimum possible frequency amount that will make this a different pitch for this violinist, therefore creating a very small number of beats. Then the DB's dynamic drops down to *piano*.

– The VLN to its right will imitate the previous action: pick up this higher-B in *forte* and very slightly raise the pitch. The previous VN will drop to *piano* and the present violin will raise it as little as possible so that it interferes with the previous higher-B. This will go on round the circle.

– Perhaps leave around 3 seconds at *forte* before passing the note to the player on your right. And leave another 3 seconds to allow the next player to pick it up exactly as it is and raise it. This means it will be at least 42 seconds before the mutated-B arrives to the DB again. This accumulative *gliss* of microtones will carry on round the circle.

– At one point the DB will 'shoot' a different note, Let's say X, in a contrasting pitch-zone, while the B is constantly mutating. The VLN to the right of the DB will then leave the B-zone and pick up the X, in the same manner as before.

– At one point, the mutated B will arrive again to the DB, who will switch again from the X to the B-zone (which by that point will probably be around D). And again, it will pass it to the VLN. After the DB changes pitch zone again, it will shoot pitch Y.

– At some point after the appearance of the Y-zone, the VC at the opposite side of the circle will play a very high pitch (suggested F), which will then be picked up by the player to its LEFT, and LOWERED.

– Players need to make sure they look at the person who they are passing the pitch to, so there is no confusion of direction. However, once you receive a cue from your right, you will stop picking up those coming from your left. This means the circle motion will eventually change direction.

– The VC is in charge of giving the new pitch material. At one point the VC should decide not to pick up a pitch from its right. Then the player to its left will also stop playing, and so on. The VLN on the VC's left will be finally left alone and will finish the piece with the last mutated-F pitch-zone.

Sara Cubarsi, February 27th 2014, Nashville

Fig 6.1. Instruction score for *Microtonal Spiral*

6.2. *8-Part Intonation Exercise*

During the exchange project, arrangements and 'arrangements of arrangements' were explored which is why I decided to microtonalize these chords from the chorale used (arranged) by Bach in his Cantata BWV60 (*Es ist Genug*), which were arranged by Berg in his Violin Concerto.

The exercise is for 8 players divided into four groups (SATB setting). The players use my discrete-glissando technique in pairs: Group 1 (Player 1 and Player 2) plays the beginning tone of their voice and then Player 1 steps up if the next note of the next chord is above (or vice-versa), by the smallest possible microtonal value. Player 2 holds the original pitch and waits until they can hear the microtonal clash clearly before reaching the same pitch as Player 1. The same action is the repeated by Player 2. The unison-clash-unison-clash is repeated until the next note is reached in unison. The other voices move in the same way. Once a pair of players reaches the next note they must wait for the other groups to reach theirs. This means the group with the shortest way to travel (the base – or those with the least sensitive aural skills!) will have to decide when the pitch has been reached. This decision is shared within the voice as there will be one of the players who will not step out of a unison at a certain point (when they belive the chord's note has been reached). As each voice reaches the note in the chord they will have to adjust to the group that has arrived before them, and so the process is repeated. The chords should be built purely using just intonation and always (highly likely) from the base (as it has the smallest distance to travel). This works as an aural and ensemble intonation exercise. Note that if a group arrives at the 'wrong unison' the whole ensemble should adjust to it. In the sections going from chord 1 to 2, and from 3 to 4, this is not likely to happen drastically as there is one voice holding the same note over the bar line. This exercise requires immense concentration: a high state of awareness and 'contemplatibility'.

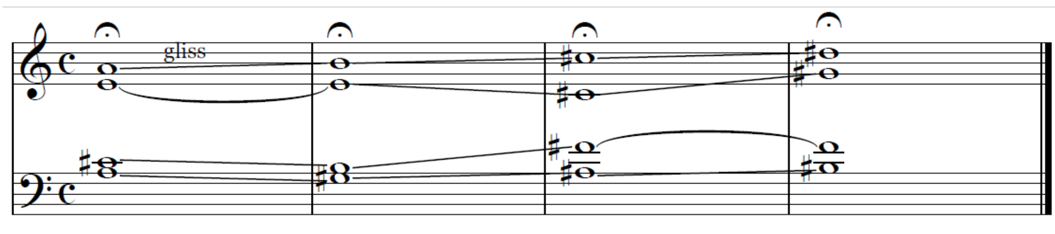


Fig 6.2. ***8-Part Intonation Exercise***

Part 7. Conclusions

It is not the mathematics that makes music sound in tune: it is the performers following leads from composers who themselves understand how to design the music so that this clarity of vibration relations results.[164]

BEN JOHNSTON

Why not let the performers take the lead? This dissertation has presented the result of such an initiative. The symbiosis of the performer, composer and theoretician has contributed with an original and trouble-free approach to microtonality. The best way to conclude is by listening to the CD and looking through the music book.

I would like to add some final remarks. My artistic research is conditioned by my personal taste which this dissertation does not attempt to objectify. This can be detected, for example, in my predilection to explore extended Pythagoreanism instead of extended JI, and this preference (statistically objectified in my summary of Seashore's studies in Part 3) is also pre-conditioned by the fact I am a 'modern' violinist. I feel that 3-limit intonation has an energizing impact on music-making (which can especially be heard in the *53TET Study* and *53TET Fugue)*, while pure thirds and sixths sound rather plain (but beautiful) to me, and pair divisions of the tone produce a less resonant sound which I experienced during the preparation of the *Three Quarter-tone Studies*.

In the composition of the *Seven Microtonal Variations* it was unintended to become detached from a tonal centre and from the modal flavour of the original chant, resulting in the use of the neutral-third (*Variation I*) through the eleventh harmonic (which gives access to a tempered quarter-tone), the seventh harmonic and higher partials (e.g., *Variation V*). I felt as if *Variation VI* failed to portray the Pythagorean language I have presented and could have been notated within a different microtonal temperament. Perhaps the fact that it was to be played at A=424Hz was an obtrusive challenge given my perfect-pitch condition.

I am aware that, in the act of performing, imprecisions are inevitable and that the system I developed also had to take that into account. Just as one can hear (or guess) the 'true' interval relationships during moderately out of tune instances in performances of Bach's solo violin works or in solo (equally tempered) piano

164 Johnston, *Maximum Clarity and other writings on music*, 88.

music, one can also hear the Pythagorean language in my studies even when there are intonational mistakes. Although the CD is not presented as a professional recording, it shows that we are capable of making distinctions between 53rd-tones (as it is for me to know exactly where the recordings went wrong!). Given the rehearsal/practice time space the results show clearly that despite the various levels of imprecision my approach to Pythagorean tuning as a microtonal resource constitutes a system of its own, which takes its most convincing form in the *53TET 2-Part Counterpoint*, opening a new tonal language to explore.

All the artistic outcome of this project shares the idea of a virtual unfretting and re-fretting of the violin's fingerboard, creating a dense and flexible sound space, and taking its most austere form in the *Microtonal Spiral* and *8-Part Intonation Exercise*.

Immediate steps following my research are the development of a tutoring method for microtonal string practice, the expansion the collaborative space (potentially including mathematicians in the theorizing of tunings systems) and further practice-based research on the possibilities offered by 53TET.

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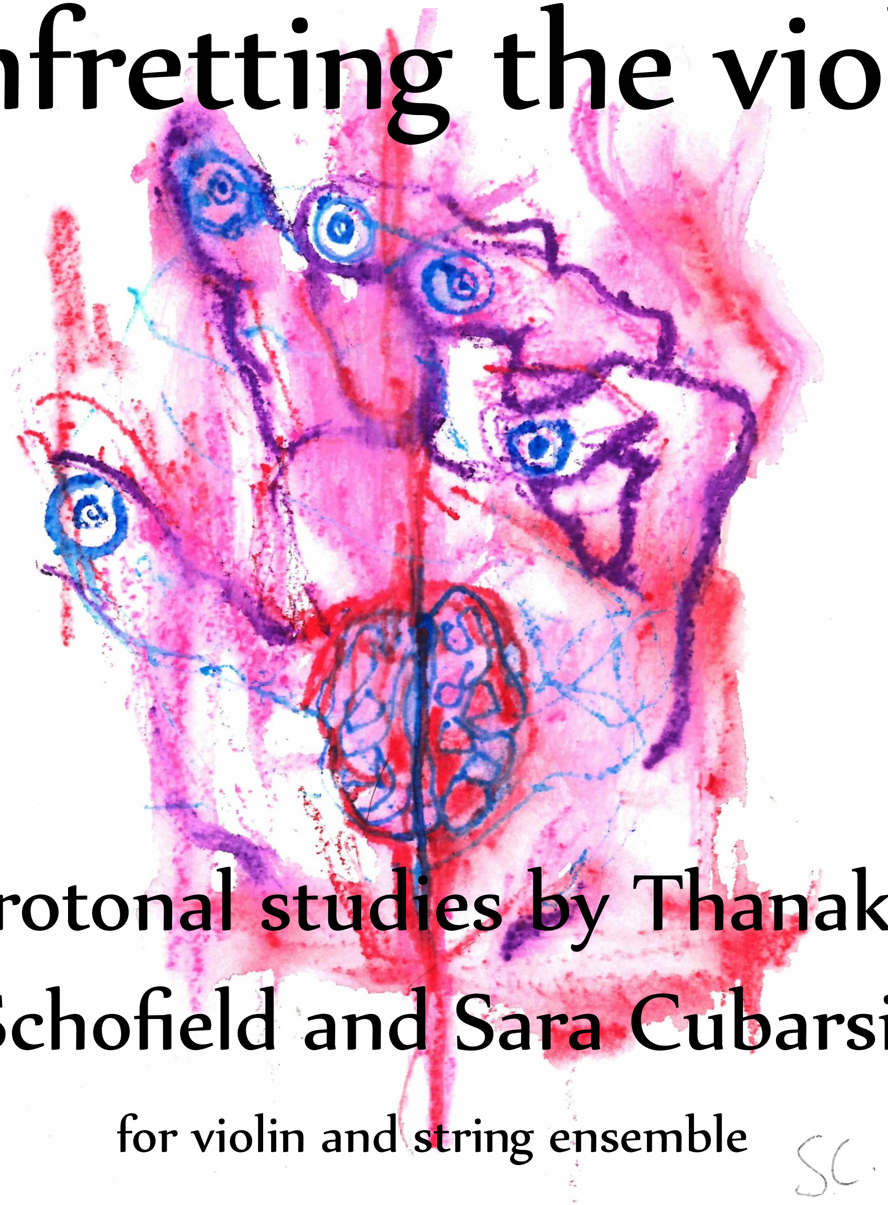


Microtonal studies by Thanakarn Schofield and Sara Cubarsi

for violin and string ensemble

2014

Three Quarter-tone Studies

Thanakarn Schofield

53TET Study

Sara Cubarsi

53TET Fugue

Sara Cubarsi

53TET 2-Part Counterpoint

Sara Cubarsi

Seven Microtonal Variations

Thanakarn Schofield and Sara Cubarsi

Microtonal Spiral

Sara Cubarsi

8-Part Intonation Exercise

Sara Cubarsi

Appendix

Three Quarter-tone Studies

𝄲 = + 50 Cents

𝄳 = - 50 Cents

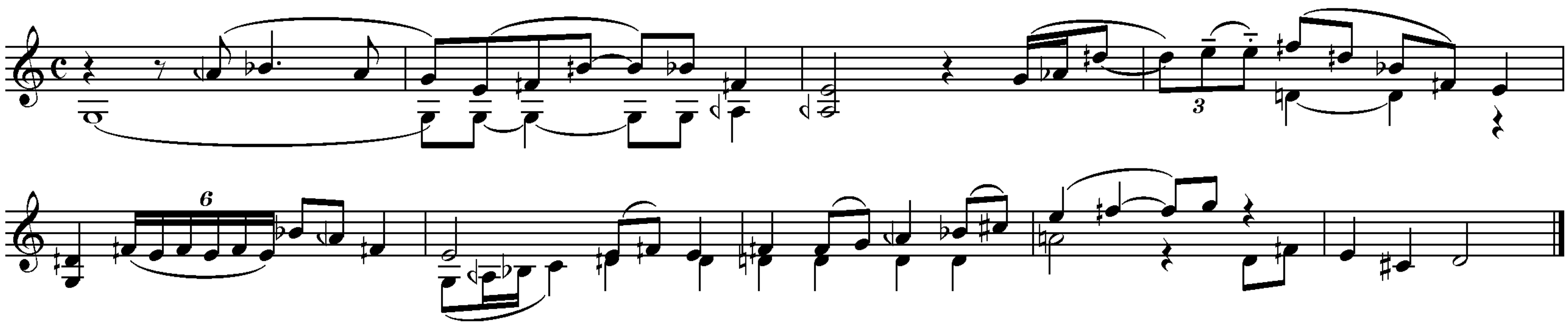
(see Appendix 1)

Thanakarn Schofield

I.

II.

III.



53TET Study

(see Appendix 2)

Sara Cubarsi

Play in 'expressive' intonation.

Aim to play all fifths and fourths pure (i.e., the *Fx* in bar 3 should be about an eight-tone higher than the open G-string). Sharps should be sharper than their enharmonically equivalent flats.

53TET Fugue

Sara Cubarsi

𝄲 = + 45 Cents	♯ = + 113 C	𝄰 = + 136 C	𝄪 = + 226 C
𝄳 = - 45 Cents	♭ = - 113 C	𝄭 = - 136 C	𝄫 = - 226 C

53TET 2-Part Counterpoint

Sara Cubarsi

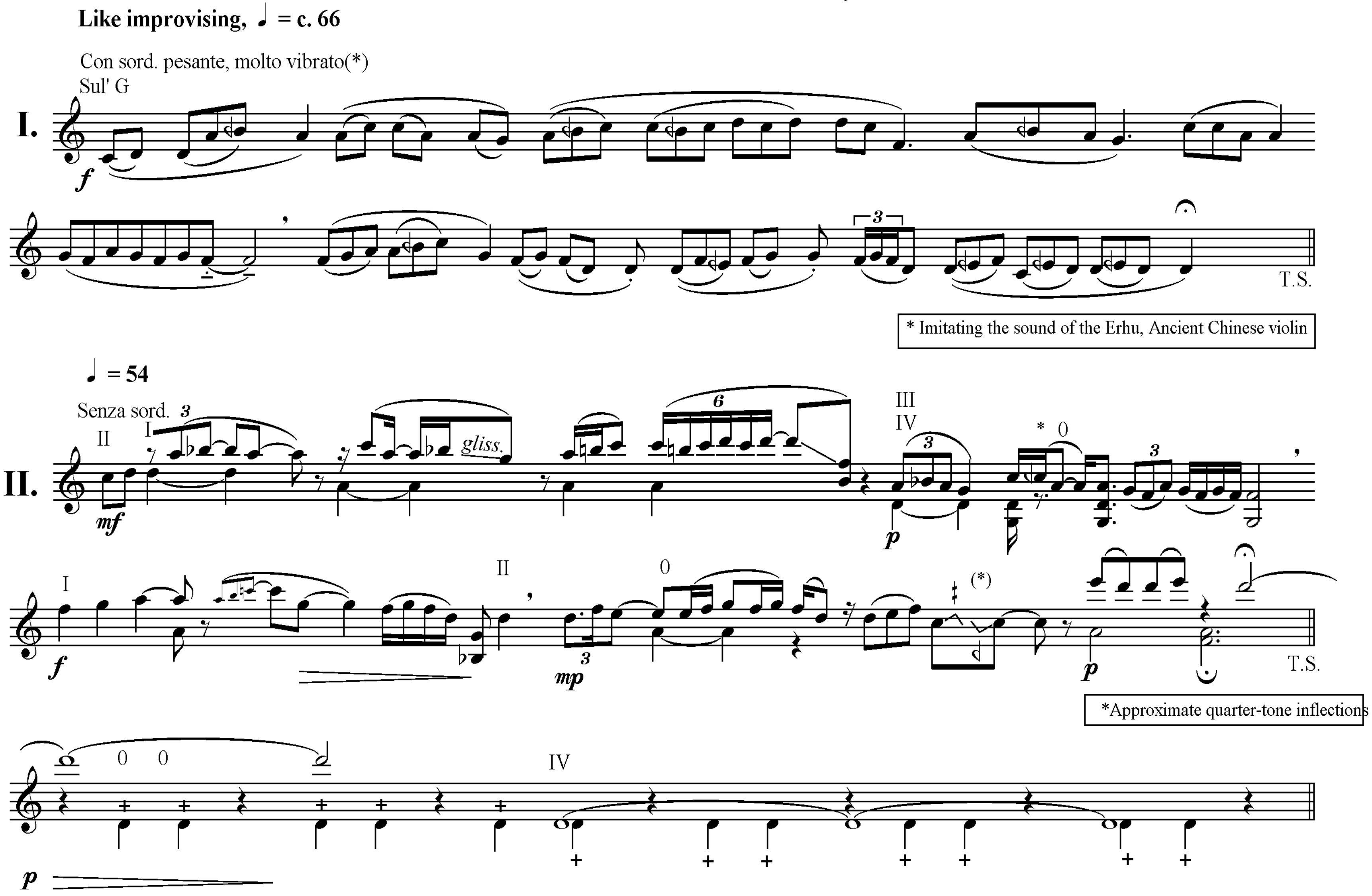
Play in 'expressive' intonation. The two groups should NOT adjust to each other.
Play without vibrato. Boxed numbers indicate the number of 53rd steps in the har-


0
17
18
19
20
21
22
23
24
gliss.
25
26
27
28
29
30
31
32
gliss.

Seven Microtonal Variations

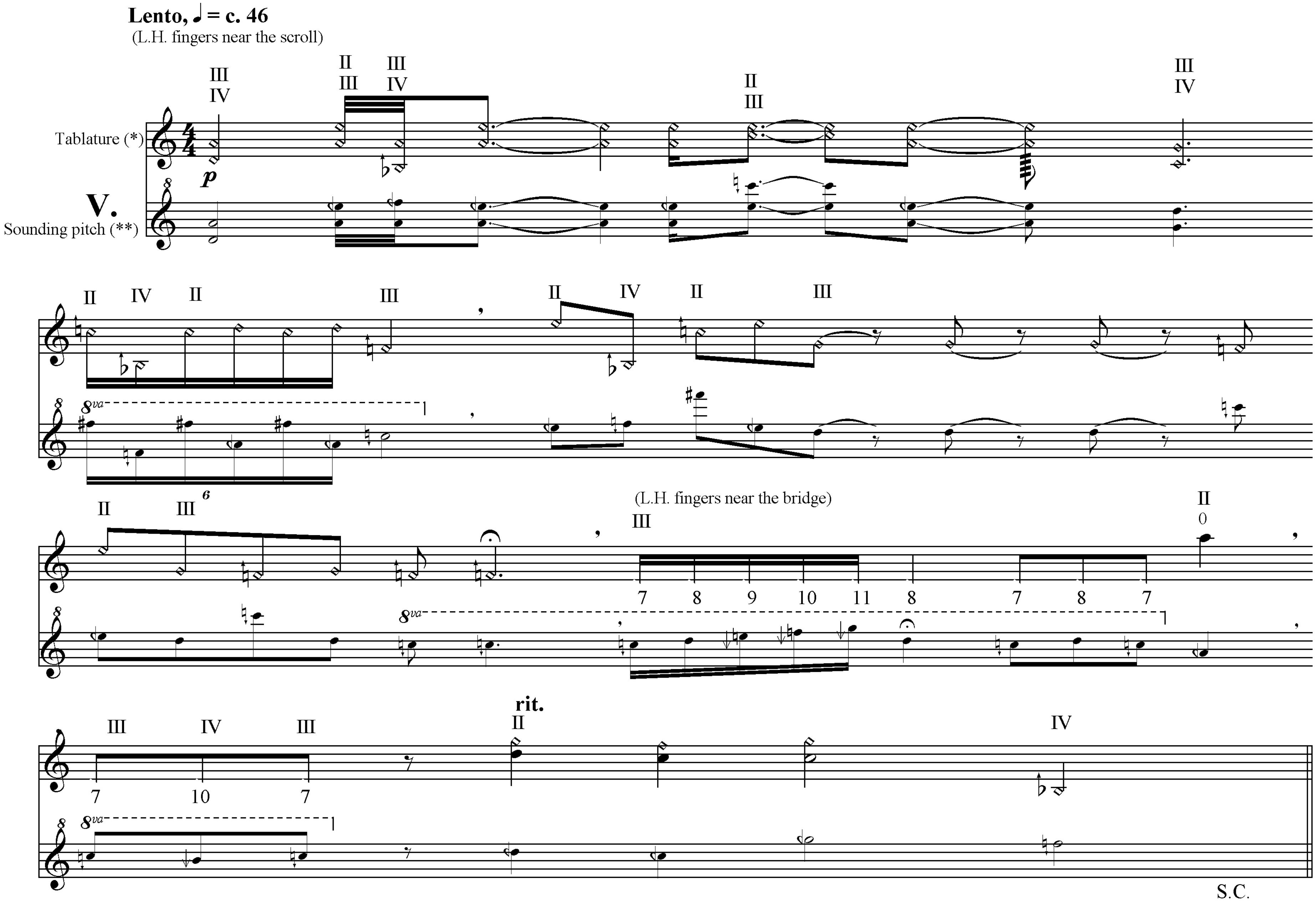
By Thanakarn Schofield and Sara Cubarsi

Scordatura tuning:
E and A strings tuned to A = 424 Hz.
D and G strings tuned to A = 442 Hz.

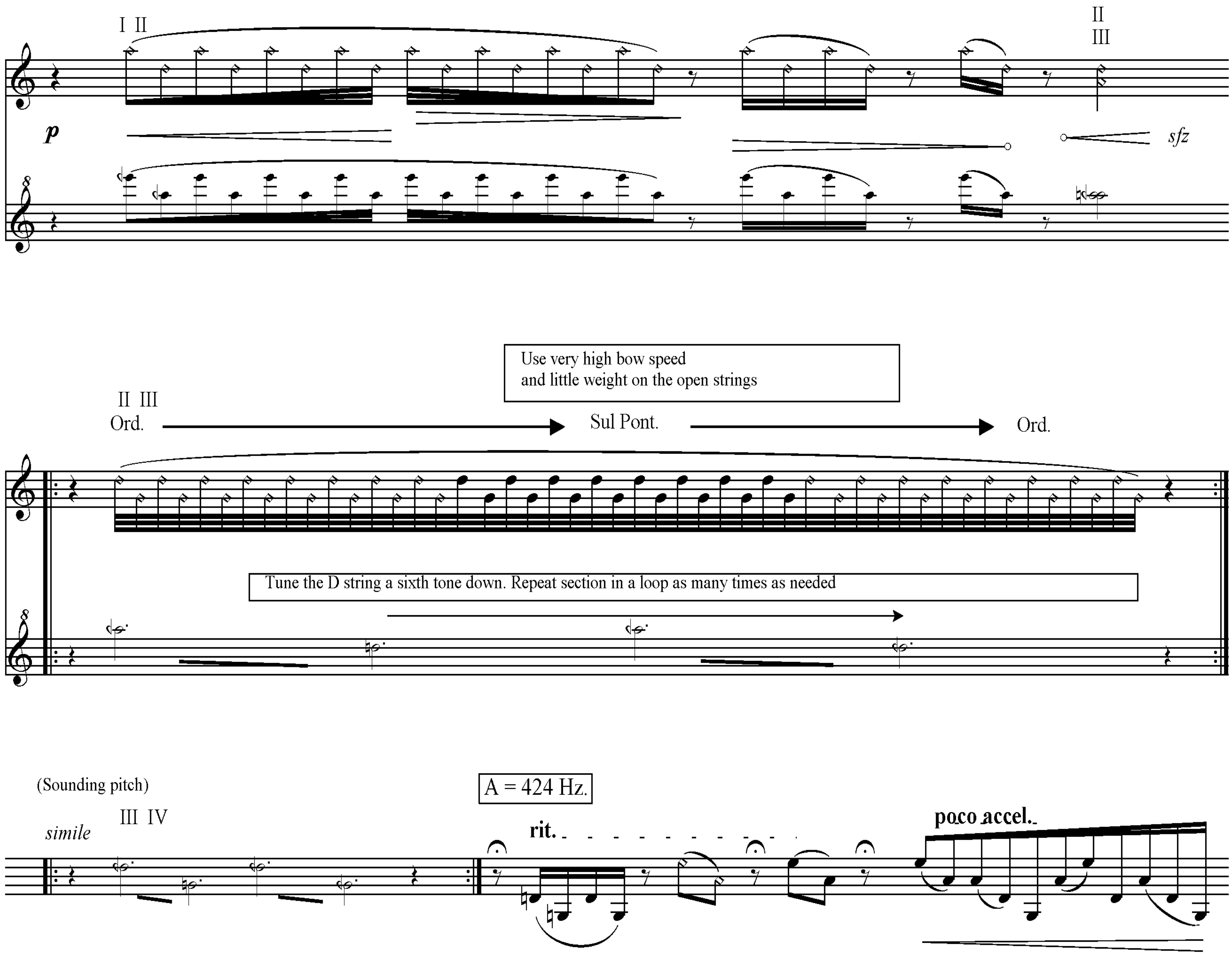


* Eighth-tone vibrato
** In strict quarter-tones

* In quarter-tones
** Wide vibrato



* Arrows attached to the accidentals indicate the placing of the pitch in tablature about a sixth tone upwards. ** Arrows attached to the accidentals indicate the pitch sounds a sixth tone lower.The quarter-tone accidental indicates the pitch sounds a 1/3-tone lower. Arrows on their own indicate the pitch sounds slighty lower.


I II
II
III
p
sfz
Use very high bow speed
and little weight on the open strings
II III
Ord.
Sul Pont.
Ord.
Tune the D string a sixth tone down. Repeat section in a loop as many times as needed
(Sounding pitch)
simile
III IV
A = 424 Hz.
rit.
poco accel.

A Tempo, molto appassionato & rubato, ♩=c.69

*

‡ / ǂ♭ = +/- 45 cents	♯ / ♭ = +/- 113c.	♯ / ♭ = +/- 136c.	× / 𝄫 = +/- 226c.

Andante

In a Baroque style (*), con sord. leggero

* The double stops are to be played in Historical Just Intonation. + amd - indicate raising or lowering the note by a Syntonic Comma with respect to a C Major scale in "Pythagorean-expressive" intonation. Freely ornamented.

THE MICROTONAL SPIRAL

For Double-bass, 5 Violins and Cello

– Sit in a circle.

– The VC opposite the DB.

– The DB starts with a B'': *forte*, tenuto, round sound and slow bow speed. The VN to its left plays the same note. The VN then raises it by the minimum possible frequency amount that will make this a different pitch for this violinist, therefore creating a very small number of beats. Then the DB's dynamic drops down to *piano*.

– The VLN to its right will imitate the previous action: pick up this higher-B in *forte* and very slightly raise the pitch. The previous VN will drop to *piano* and the present violin will raise it as little as possible so that it interferes with the previous higher-B. This will go on round the circle.

– Perhaps leave around 3 seconds at *forte* before passing the note to the player on your right. And leave another 3 seconds to allow the next player to pick it up exactly as it is and raise it. This means it will be at least 42 seconds before the mutated-B arrives to the DB again. This accumulative *gliss* of microtones will carry on round the circle.

– At one point the DB will 'shoot' a different note, Let's say X, in a contrasting pitch-zone, while the B is constantly mutating. The VLN to the right of the DB will then leave the B-zone and pick up the X, in the same manner as before.

– At one point, the mutated B will arrive again to the DB, who will switch again from the X to the B-zone (which by that point will probably be around D). And again, it will pass it to the VLN. After the DB changes pitch zone again, it will shoot pitch Y.

– At some point after the appearance of the Y-zone, the VC at the opposite side of the circle will play a very high pitch (suggested F), which will then be picked up by the player to its LEFT, and LOWERED.

– Players need to make sure they look at the person who they are passing the pitch to, so there is no confusion of direction. However, once you receive a cue from your right, you will stop picking up those coming from your left. This means the circle motion will eventually change direction.

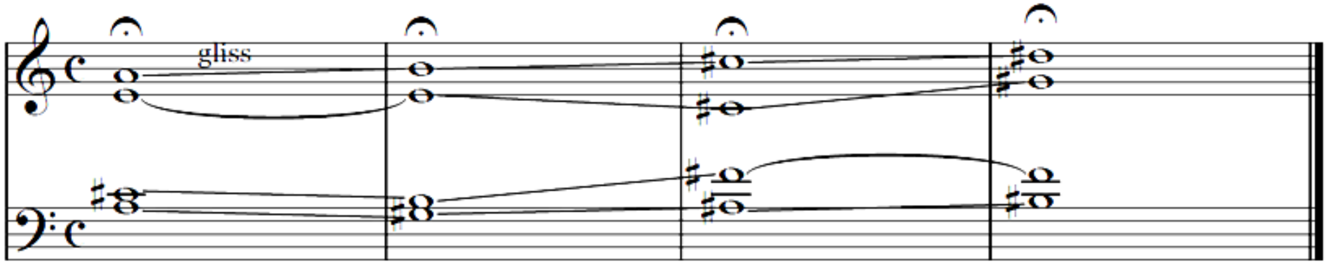
– The VC is in charge of giving the new pitch material. At one point the VC should decide not to pick up a pitch from its right. Then the player to its left will also stop playing, and so on. The VLN on the VC's left will be finally left alone and will finish the piece with the last mutated-F pitch-zone.

Sara Cubarsi, February 27th 2014, Nashville

8-Part Intonation Study

for strings

'Es ist Genug', Arr. Sara Cubarsi



Performance instructions:

-Two players per 'voice'

-Each pair of players should move towards the following note with the discrete-glissando* technique

-A pair of players (with the shortest interval leap) will decide when they have reached the notated pitch and meet at the unison, holding the pitch during the pauses.

-The rest will reach their respective notes and adjust in order to built the notated chord. Once a chord is tuned, the same process will be repeated toward the following one.

*Technique where the glissando is shared. One player plays a note and the another player plays its unison and raises or lowers its pitch by the least frequency amount to make it different (amount will vary depending on the performer) and, then, passes it on to another player who repeats the action.

Appendix

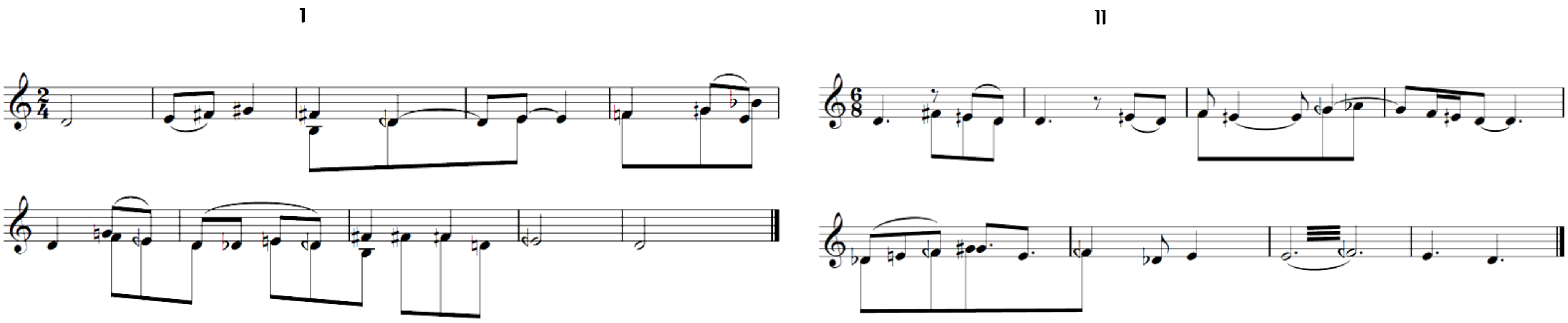
1. Three Quarter-tone Studies

The studies are based on different microtonal tetrachords (A). The quarter-tones can be approached intuitively as ‘leading tones of half tones’ or as an equal division of a larger interval (B). The notes with a lower stem indicate the interval that the quarter-tone ‘divides’ (C).

A

B

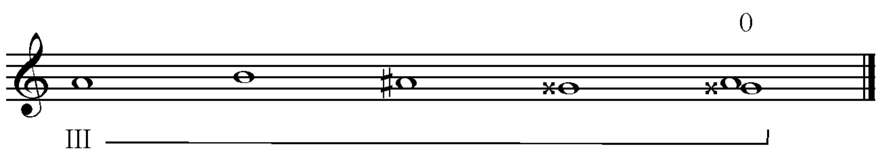
C



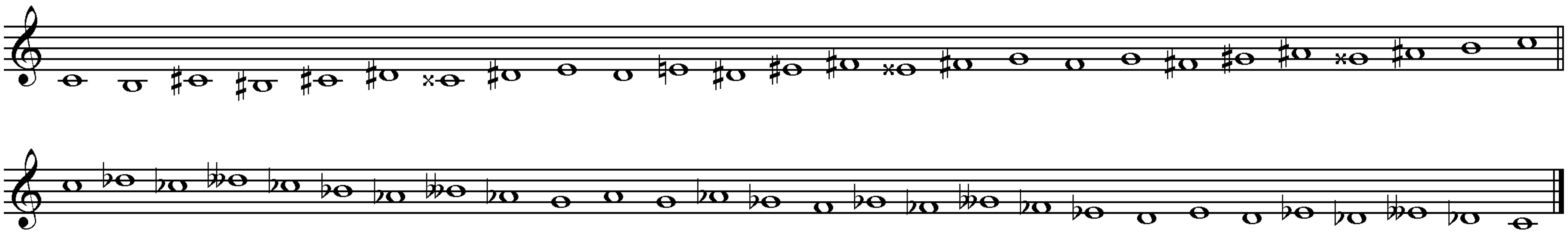
2. 53-Tone Equal Temperament: the basics

In a cycle of 53 fifths the last fifth surpasses the generating tone by approximately 4 Cents. 53TET tempers out this cycle by 0.68 Cents, therefore the fifths remain practically pure. Sharps are obtained by adding fifths, and flats by subtracting fifths. My proposed notation shown in *53TET Fugue* replaces all the other tones which need more than a double-flat or double-sharp sign with microtonal accidentals.

The 'Pythagorean comma' is the smallest interval in 53TET. The comma can be heard by comparing two enharmonic notes, e.g., Gx with the open A-string. There should be an interference of 6 beats per second (A=442Hz and Gx=447Hz):



Play the following scale and listen out for the different response the instrument has between the ascending scale and the descending scale:



The following exercise plays around 4 different 53TET intervals. Numbers indicate the number of 53rd steps or 'Pythagorean commas' between two pitches. Memorize the sound of the first bar and repeat the exercise in a loop as shown below: